Forecasting

CHAPTER 4

CHAPTER OUTLINE

GLOBAL COMPANY PROFILE: *Walt Disney Parks & Resorts*

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Alaska Airlines

GLOBAL COMPANY PROFILE
Walt Disney Parks & Resorts

Forecasting Provides a Competitive Advantage for Disney

When it comes to the world's most respected global brands, Walt Disney Parks & Resorts is a visible leader. Although the monarch of this magic kingdom is no man but a mouse—Mickey Mouse—it's CEO Robert Iger who daily manages the entertainment giant.

Disney's global portfolio includes Shanghai Disney (2016), Hong Kong Disneyland (2005), Disneyland Paris (1992), and Tokyo Disneyland (1983). But it is Walt Disney World Resort (in Florida) and Disneyland Resort (in California) that drive profits in this $50 billion corporation, which is ranked in the top 100 in both the *Fortune* 500 and *Financial Times* Global 500.

Travelshots/Peter Phipp/Travelshots.com/Alamy

Donald Duck, Goofy, and Mickey Mouse provide the public image of Disney to the world. Forecasts drive the work schedules of 72,000 cast members working at Walt Disney World Resort near Orlando.

Revenues at Disney are all about people—how many visit the parks and how they spend money while there. When Iger receives a daily report from his four theme parks and two water parks near Orlando, the report contains only two numbers: the *forecast* of yesterday's attendance at the parks (Magic Kingdom, Epcot, Disney's Animal Kingdom, Disney-Hollywood Studios, Typhoon Lagoon, and Blizzard Beach) and the *actual* attendance. An error close to zero is expected. Iger takes his forecasts very seriously.

The forecasting team at Walt Disney World Resort doesn't just do a daily prediction, however, and Iger is not its only customer. The team also provides daily, weekly, monthly, annual, and 5-year forecasts to the labor management, maintenance, operations, finance, and park scheduling departments. Forecasters use judgmental models, econometric models, moving-average models, and regression analysis.

The giant sphere is the symbol of Epcot, one of Disney's four Orlando parks, for which forecasts of meals, lodging, entertainment, and transportation must be made. This Disney monorail moves guests among parks and the 28 hotels on the massive 47-square-mile property (about the size of San Francisco and twice the size of Manhattan).

Nicolas Chan/Alamy

Kevin Fleming/Corbis

A daily forecast of attendance is made by adjusting Disney's annual operating plan for weather forecasts, the previous day's crowds, conventions, and seasonal variations. One of the two water parks at Walt Disney World Resort, Typhoon Lagoon, is shown here.

Melvyn Longhurst/Corbis

Cinderella's iconic castle is a focal point for meeting up with family and friends in the massive park. The statue of Walt Disney greets visitors to the open plaza.

With 20% of Walt Disney World Resort's customers coming from outside the United States, its economic model includes such variables as gross domestic product (GDP), cross-exchange rates, and arrivals into the U.S. Disney also uses 35 analysts and 70 field people to survey 1 million people each year. The surveys, administered to guests at the parks and its 20 hotels, to employees, and to travel industry professionals, examine future travel plans and experiences at the parks. This helps forecast not only attendance but also behavior at each ride (e.g., how long people will wait, how many times they will ride). Inputs to the monthly forecasting model include airline specials, speeches by the chair of the Federal Reserve, and Wall Street trends. Disney even monitors 3,000 school districts inside and outside the U.S. for holiday/vacation schedules. With this approach, Disney's 5-year attendance forecast yields just a 5% error on average. Its annual forecasts have a 0% to 3% error.

Attendance forecasts for the parks drive a whole slew of management decisions. For example, capacity on any day can be increased by opening at 8 A.M. instead of the usual 9 A.M., by opening more shows or rides, by adding more food/beverage carts (9 million hamburgers and 50 million Cokes are sold per year!), and by bringing in more employees (called "cast members"). Cast members are scheduled in 15-minute intervals throughout the parks for flexibility. Demand can be managed by limiting the number of guests admitted to the parks, with the "FAST PASS" reservation system, and by shifting crowds from rides to more street parades.

dmac/Alamy

Forecasts are critical to making sure rides are not overcrowded. Disney is good at "managing demand" with techniques such as adding more street activities to reduce long lines for rides. On slow days, Disney calls fewer cast members to work.

At Disney, forecasting is a key driver in the company's success and competitive advantage.

LEARNING OBJECTIVES

LO 4.1 ***Understand*** the three time horizons and which models apply for each 108

LO 4.2 ***Explain*** when to use each of the four qualitative models 111

LO 4.3 ***Apply*** the naive, moving-average, exponential smoothing, and trend methods 113

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What Is Forecasting?

Every day, managers like those at Disney make decisions without knowing what will happen in the future. They order inventory without knowing what sales will be, purchase new equipment despite uncertainty about demand for products, and make investments without knowing what profits will be. Managers are always trying to make better estimates of what will happen in the future in the face of uncertainty. Making good estimates is the main purpose of forecasting.

STUDENT TIP An increasingly complex world economy makes forecasting challenging.

In this chapter, we examine different types of forecasts and present a variety of forecasting models. Our purpose is to show that there are many ways for managers to forecast. We also provide an overview of business sales forecasting and describe how to prepare, monitor, and judge the accuracy of a forecast. Good forecasts are an *essential* part of efficient service and manufacturing operations.

Forecasting
The art and science of predicting future events.

Forecasting is the art and science of predicting future events. Forecasting may involve taking historical data (such as past sales) and projecting them into the future with a mathematical model. It may be a subjective or an intuitive prediction (e.g., "this is a great new product and will sell 20% more than the old one"). It may be based on demand-driven data, such as customer plans to purchase, and projecting them into the future. Or the forecast may involve a combination of these, that is, a mathematical model adjusted by a manager's good judgment.

As we introduce different forecasting techniques in this chapter, you will see that there is seldom one superior method. Forecasts may be influenced by a product's position in its life cycle—whether sales are in an introduction, growth, maturity, or decline stage. Other products can be influenced by the demand for a related product—for example, navigation systems may track with new car sales. Because there are limits to what can be expected from forecasts, we develop error measures. Preparing and monitoring forecasts can also be costly and time consuming.

Few businesses, however, can afford to avoid the process of forecasting by just waiting to see what happens and then taking their chances. Effective planning in both the short run and long run depends on a forecast of demand for the company's products.

Forecasting Time Horizons

LO 4.1 *Understand* the three time horizons and which models apply for each

A forecast is usually classified by the *future time horizon* that it covers. Time horizons fall into three categories:

1. *Short-range forecast:* This forecast has a time span of up to 1 year but is generally less than 3 months. It is used for planning purchasing, job scheduling, workforce levels, job assignments, and production levels.
2. *Medium-range forecast:* A medium-range, or intermediate, forecast generally spans from 3 months to 3 years. It is useful in sales planning, production planning and budgeting, cash budgeting, and analysis of various operating plans.
3. *Long-range forecast:* Generally 3 years or more in time span, long-range forecasts are used in planning for new products, capital expenditures, facility location or expansion, and research and development.

Medium- and long-range forecasts are distinguished from short-range forecasts by three features:

1. First, intermediate and long-range forecasts *deal with more comprehensive issues* supporting management decisions regarding planning and products, plants, and processes. Implementing some facility decisions, such as GM's decision to open a new Brazilian manufacturing plant, can take 5 to 8 years from inception to completion.
2. Second, short-term forecasting usually *employs different methodologies* than longer-term forecasting. Mathematical techniques, such as moving averages, exponential smoothing, and trend extrapolation (all of which we shall examine shortly), are common to short-run projections. Broader, *less* quantitative methods are useful in predicting such issues as whether a new product, like the optical disk recorder, should be introduced into a company's product line.
3. Finally, as you would expect, short-range forecasts *tend to be more accurate* than longer-range forecasts. Factors that influence demand change every day. Thus, as the time horizon lengthens, it is likely that forecast accuracy will diminish. It almost goes without saying, then, that sales forecasts must be updated regularly to maintain their value and integrity. After each sales period, forecasts should be reviewed and revised.

Types of Forecasts

Organizations use three major types of forecasts in planning future operations:

1. **Economic forecasts** address the business cycle by predicting inflation rates, money supplies, housing starts, and other planning indicators.
2. **Technological forecasts** are concerned with rates of technological progress, which can result in the birth of exciting new products, requiring new plants and equipment.
3. **Demand forecasts** are projections of demand for a company's products or services. Forecasts drive decisions, so managers need immediate and accurate information about real demand. They need *demand-driven forecasts*, where the focus is on rapidly identifying and tracking customer desires. These forecasts may use recent point-of-sale (POS) data, retailer-generated reports of customer preferences, and any other information that will help to forecast with the most current data possible. Demand-driven forecasts drive a company's production, capacity, and scheduling systems and serve as inputs to financial, marketing, and personnel planning. In addition, the payoff in reduced inventory and obsolescence can be huge.

Economic forecasts
Planning indicators that are valuable in helping organizations prepare medium- to long-range forecasts.

Technological forecasts
Long-term forecasts concerned with the rates of technological progress.

Demand forecasts
Projections of a company's sales for each time period in the planning horizon.

Economic and technological forecasting are specialized techniques that may fall outside the role of the operations manager. The emphasis in this chapter will therefore be on demand forecasting.

The Strategic Importance of Forecasting

Good forecasts are of critical importance in all aspects of a business: *The forecast is the only estimate of demand until actual demand becomes known.* Forecasts of demand therefore drive decisions in many areas. Let's look at the impact of product demand forecast on three activities: (1) supply-chain management, (2) human resources, and (3) capacity.

Supply-Chain Management

Good supplier relations and the ensuing advantages in product innovation, cost, and speed to market depend on accurate forecasts. Here are just three examples:

- Apple has built an effective global system where it controls nearly every piece of the supply chain, from product design to retail store. With rapid communication and accurate data shared up and down the supply chain, innovation is enhanced, inventory costs are reduced, and speed to market is improved. Once a product goes on sale, Apple tracks demand by the hour for each store and adjusts production forecasts daily. At Apple, forecasts for its supply chain are a strategic weapon.

- Toyota develops sophisticated car forecasts with input from a variety of sources, including dealers. But forecasting the demand for accessories such as navigation systems, custom wheels, spoilers, and so on is particularly difficult. And there are over 1,000 items that vary by model and color. As a result, Toyota not only reviews reams of data with regard to vehicles that have been built and wholesaled but also looks in detail at vehicle forecasts before it makes judgments about the future accessory demand. When this is done correctly, the result is an efficient supply chain and satisfied customers.
- Walmart collaborates with suppliers such as Sara Lee and Procter & Gamble to make sure the right item is available at the right time in the right place and at the right price. For instance, in hurricane season, Walmart's ability to analyze 700 million store–item combinations means it can forecast that not only flashlights but also Pop-Tarts and beer sell at seven times the normal demand rate. These forecasting systems are known as *collaborative planning, forecasting, and replenishment* (CPFR). They combine the intelligence of multiple supply-chain partners. The goal of CPFR is to create significantly more accurate information that can power the supply chain to greater sales and profits.

Human Resources

Hiring, training, and laying off workers all depend on anticipated demand. If the human resources department must hire additional workers without warning, the amount of training declines, and the quality of the workforce suffers. A large Louisiana chemical firm almost lost its biggest customer when a quick expansion to around-the-clock shifts led to a total breakdown in quality control on the second and third shifts.

Capacity

When capacity is inadequate, the resulting shortages can lead to loss of customers and market share. This is exactly what happened to Nabisco when it underestimated the huge demand for its new Snackwell Devil's Food Cookies. Even with production lines working overtime, Nabisco could not keep up with demand, and it lost customers. Nintendo faced this problem when its Wii was introduced and exceeded all forecasts for demand. Amazon made the same error with its Kindle. On the other hand, when excess capacity exists, costs can skyrocket.

Seven Steps in the Forecasting System

Forecasting follows seven basic steps. We use Disney World, the focus of this chapter's *Global Company Profile*, as an example of each step:

1. *Determine the use of the forecast:* Disney uses park attendance forecasts to drive decisions about staffing, opening times, ride availability, and food supplies.
2. *Select the items to be forecasted:* For Disney World, there are six main parks. A forecast of daily attendance at each is the main number that determines labor, maintenance, and scheduling.
3. *Determine the time horizon of the forecast:* Is it short, medium, or long term? Disney develops daily, weekly, monthly, annual, and 5-year forecasts.
4. *Select the forecasting model(s):* Disney uses a variety of statistical models that we shall discuss, including moving averages, econometrics, and regression analysis. It also employs judgmental, or nonquantitative, models.
5. *Gather the data needed to make the forecast:* Disney's forecasting team employs 35 analysts and 70 field personnel to survey 1 million people/businesses every year. Disney also uses a firm called Global Insights for travel industry forecasts and gathers data on exchange rates, arrivals into the U.S., airline specials, Wall Street trends, and school vacation schedules.

6. *Make the forecast.*
7. *Validate and implement the results:* At Disney, forecasts are reviewed daily at the highest levels to make sure that the model, assumptions, and data are valid. Error measures are applied; then the forecasts are used to schedule personnel down to 15-minute intervals.

These seven steps present a systematic way of initiating, designing, and implementing a forecasting system. When the system is to be used to generate forecasts regularly over time, data must be routinely collected. Then actual computations are usually made by computer.

Regardless of the system that firms like Disney use, each company faces several realities:

- Outside factors that we cannot predict or control often impact the forecast.
- Most forecasting techniques assume that there is some underlying stability in the system. Consequently, some firms automate their predictions using computerized forecasting software, then closely monitor only the product items whose demand is erratic.
- Both product family and aggregated forecasts are more accurate than individual product forecasts. Disney, for example, aggregates daily attendance forecasts by park. This approach helps balance the over- and underpredictions for each of the six attractions.

Forecasting Approaches

There are two general approaches to forecasting, just as there are two ways to tackle all decision modeling. One is a quantitative analysis; the other is a qualitative approach. **Quantitative forecasts** use a variety of mathematical models that rely on historical data and/or associative variables to forecast demand. Subjective or **qualitative forecasts** incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system in reaching a forecast. Some firms use one approach and some use the other. In practice, a combination of the two is usually most effective.

Quantitative forecasts
Forecasts that employ mathematical modeling to forecast demand.

Qualitative forecasts
Forecasts that incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system.

Overview of Qualitative Methods

In this section, we consider four different *qualitative* forecasting techniques:

1. **Jury of executive opinion**: Under this method, the opinions of a group of high-level experts or managers, often in combination with statistical models, are pooled to arrive at a group estimate of demand. Bristol-Myers Squibb Company, for example, uses 220 well-known research scientists as its jury of executive opinion to get a grasp on future trends in the world of medical research.
2. **Delphi method**: There are three different types of participants in the Delphi method: decision makers, staff personnel, and respondents. Decision makers usually consist of a group of 5 to 10 experts who will be making the actual forecast. Staff personnel assist decision makers by preparing, distributing, collecting, and summarizing a series of questionnaires and survey results. The respondents are a group of people, often located in different places, whose judgments are valued. This group provides inputs to the decision makers before the forecast is made.

   The state of Alaska, for example, has used the Delphi method to develop its long-range economic forecast. A large part of the state's budget is derived from the million-plus barrels of oil pumped daily through a pipeline at Prudhoe Bay. The large Delphi panel of experts had to represent all groups and opinions in the state and all geographic areas.
3. **Sales force composite**: In this approach, each salesperson estimates what sales will be in his or her region. These forecasts are then reviewed to ensure that they are realistic. Then they are combined at the district and national levels to reach an overall forecast. A variation of this approach occurs at Lexus, where every quarter Lexus dealers have a "make meeting." At this meeting, they talk about what is selling, in what colors, and with what options, so the factory knows what to build.
4. **Market survey**: This method solicits input from customers or potential customers regarding future purchasing plans. It can help not only in preparing a forecast but also in improving

Jury of executive opinion
A forecasting technique that uses the opinion of a small group of high-level managers to form a group estimate of demand.

Delphi method
A forecasting technique using a group process that allows experts to make forecasts.

LO 4.2 *Explain* when to use each of the four qualitative models

Sales force composite
A forecasting technique based on salespersons' estimates of expected sales.

Market survey
A forecasting method that solicits input from customers or potential customers regarding future purchasing plans.

product design and planning for new products. The consumer market survey and sales force composite methods can, however, suffer from overly optimistic forecasts that arise from customer input.

Overview of Quantitative Methods[1]

Five quantitative forecasting methods, all of which use historical data, are described in this chapter. They fall into two categories:

1. Naive approach
2. Moving averages
3. Exponential smoothing
4. Trend projection

} **Time-series models**

5. Linear regression } **Associative model**

Time series
A forecasting technique that uses a series of past data points to make a forecast.

Time-Series Models Time-series models predict on the assumption that the future is a function of the past. In other words, they look at what has happened over a period of time and use a series of past data to make a forecast. If we are predicting sales of lawn mowers, we use the past sales for lawn mowers to make the forecasts.

Associative Models Associative models, such as linear regression, incorporate the variables or factors that might influence the quantity being forecast. For example, an associative model for lawn mower sales might use factors such as new housing starts, advertising budget, and competitors' prices.

STUDENT TIP
Here is the meat of this chapter. We now show you a wide variety of models that use time-series data.

Time-Series Forecasting

A time series is based on a sequence of evenly spaced (weekly, monthly, quarterly, and so on) data points. Examples include weekly sales of Nike Air Jordans, quarterly earnings reports of Microsoft stock, daily shipments of Coors beer, and annual consumer price indices. Forecasting time-series data implies that future values are predicted *only* from past values and that other variables, no matter how potentially valuable, may be ignored.

Decomposition of a Time Series

Analyzing time series means breaking down past data into components and then projecting them forward. A time series has four components:

1. *Trend* is the gradual upward or downward movement of the data over time. Changes in income, population, age distribution, or cultural views may account for movement in trend.
2. *Seasonality* is a data pattern that repeats itself after a period of days, weeks, months, or quarters. There are six common seasonality patterns:

STUDENT TIP
The peak "seasons" for sales of Frito-Lay chips are the Super Bowl, Memorial Day, Labor Day, and the Fourth of July.

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	$4–4\frac{1}{2}$
Month	Day	28–31
Year	Quarter	4
Year	Month	12
Year	Week	52

Restaurants and barber shops, for example, experience weekly seasons, with Saturday being the peak of business. See the *OM in Action* box "Forecasting at Olive Garden." Beer distributors forecast yearly patterns, with monthly seasons. Three "seasons"—May, July, and September—each contain a big beer-drinking holiday.

OM in Action Forecasting at Olive Garden

It's Friday night in the college town of Gainesville, Florida, and the local Olive Garden restaurant is humming. Customers may wait an average of 30 minutes for a table, but they can sample new wines and cheeses and admire scenic paintings of Italian villages on the Tuscan-style restaurant's walls. Then comes dinner with portions so huge that many people take home a doggie bag. The typical bill: under $15 per person.

Crowds flock to the Darden restaurant chain's Olive Garden, Seasons 52, and Bahama Breeze for value and consistency—*and* they get it.

Every night, Darden's computers crank out forecasts that tell store managers what demand to anticipate the next day. The forecasting software generates a total meal forecast and breaks that down into specific menu items. The system tells a manager, for instance, that if 625 meals will be served the next day, "you will serve these items in these quantities. So before you go home, pull 25 pounds of shrimp and 30 pounds of crab out, and tell your operations people to prepare 42 portion packs of chicken, 75 scampi dishes, 8 stuffed flounders, and so on." Managers often fine-tune the quantities based on local conditions, such as weather or a convention, but they know what their customers are going to order.

Bob Pardue - Signs/Alamy

By relying on demand history, the forecasting system has cut millions of dollars of waste out of the system. The forecast also reduces labor costs by providing the necessary information for improved scheduling. Labor costs decreased almost a full percent in the first year, translating into additional millions in savings for the Darden chain. In the low-margin restaurant business, every dollar counts.

Sources: InformationWeek (April 1, 2014); *USA Today* (Oct. 13, 2014); and *FastCompany* (July-August 2009).

3. *Cycles* are patterns in the data that occur every several years. They are usually tied into the business cycle and are of major importance in short-term business analysis and planning. Predicting business cycles is difficult because they may be affected by political events or by international turmoil.
4. *Random variations* are "blips" in the data caused by chance and unusual situations. They follow no discernible pattern, so they cannot be predicted.

LO 4.3 *Apply* the naive, moving-average, exponential smoothing, and trend methods

Figure 4.1 illustrates a demand over a 4-year period. It shows the average, trend, seasonal components, and random variations around the demand curve. The average demand is the sum of the demand for each period divided by the number of data periods.

Naive Approach

The simplest way to forecast is to assume that demand in the next period will be equal to demand in the most recent period. In other words, if sales of a product—say, Nokia cell phones—were 68 units in January, we can forecast that February's sales will also be 68 phones.

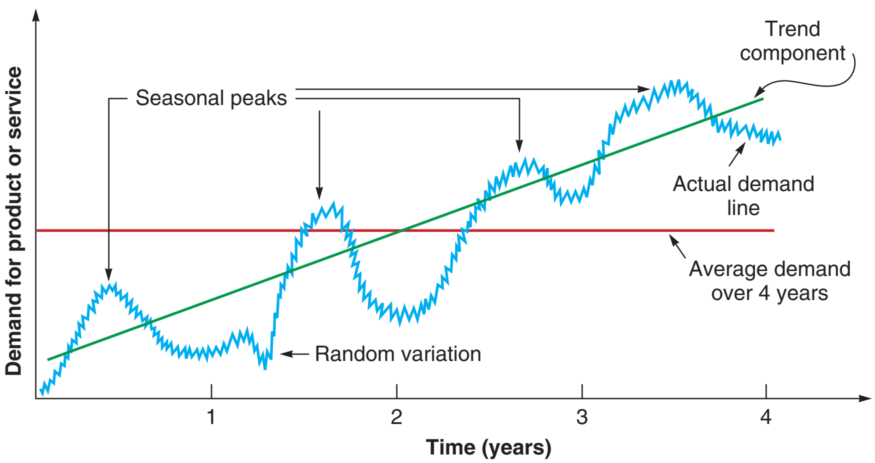


Figure **4.1**

Demand Charted over 4 Years, with a Growth Trend and Seasonality Indicated

STUDENT TIP

Forecasting is easy when demand is stable. But with trend, seasonality, and cycles considered, the job is a lot more interesting.

Naive approach
A forecasting technique that assumes that demand in the next period is equal to demand in the most recent period.

Does this make any sense? It turns out that for some product lines, this **naive approach** is the most cost-effective and efficient objective forecasting model. At least it provides a starting point against which more sophisticated models that follow can be compared.

Moving Averages

Moving averages
A forecasting method that uses an average of the *n* most recent periods of data to forecast the next period.

A **moving-average** forecast uses a number of historical actual data values to generate a forecast. Moving averages are useful *if we can assume that market demands will stay fairly steady over time*. A 4-month moving average is found by simply summing the demand during the past 4 months and dividing by 4. With each passing month, the most recent month's data are added to the sum of the previous 3 months' data, and the earliest month is dropped. This practice tends to smooth out short-term irregularities in the data series.

Mathematically, the simple moving average (which serves as an estimate of the next period's demand) is expressed as:

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n} \tag{4-1}$$

where n is the number of periods in the moving average—for example, 4, 5, or 6 months, respectively, for a 4-, 5-, or 6-period moving average.

Example 1 shows how moving averages are calculated.

Example 1

DETERMINING THE MOVING AVERAGE

Donna's Garden Supply wants a 3-month moving-average forecast, including a forecast for next January, for shed sales.

APPROACH ▶ Storage shed sales are shown in the middle column of the following table. A 3-month moving average appears on the right.

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\frac{2}{3}$
May	19	$(12 + 13 + 16)/3 = 13\frac{2}{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\frac{1}{3}$
August	30	$(19 + 23 + 26)/3 = 22\frac{2}{3}$
September	28	$(23 + 26 + 30)/3 = 26\frac{1}{3}$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\frac{1}{3}$
December	14	$(28 + 18 + 16)/3 = 20\frac{2}{3}$

SOLUTION ▶ The forecast for December is $20\frac{2}{3}$. To project the demand for sheds in the coming January, we sum the October, November, and December sales and divide by 3: January forecast $= (18 + 16 + 14)/3 = 16$.

INSIGHT ▶ Management now has a forecast that averages sales for the last 3 months. It is easy to use and understand.

LEARNING EXERCISE ▶ If actual sales in December were 18 (rather than 14), what is the new January forecast? [Answer: $17\frac{1}{3}$.]

RELATED PROBLEMS ▶ 4.1a, 4.2b, 4.5a, 4.6, 4.8a, b, 4.10a, 4.13b, 4.15, 4.33 (4.35, 4.38 are available in MyOMLab)

EXCEL OM Data File **Ch04Ex1.xls** can be found in MyOMLab.

ACTIVE MODEL 4.1 This example is further illustrated in Active Model 4.1 in MyOMLab.

When a detectable trend or pattern is present, *weights* can be used to place more emphasis on recent values. This practice makes forecasting techniques more responsive to changes because more recent periods may be more heavily weighted. Choice of weights is somewhat arbitrary because there is no set formula to determine them. Therefore, deciding which weights to use requires some experience. For example, if the latest month or period is weighted too heavily, the forecast may reflect a large unusual change in the demand or sales pattern too quickly.

A weighted moving average may be expressed mathematically as:

$$\text{Weighted moving average} = \frac{\sum((\text{Weight for period } n)(\text{Demand in period } n))}{\sum \text{Weights}} \tag{4-2}$$

Example 2 shows how to calculate a weighted moving average.

Example 2

DETERMINING THE WEIGHTED MOVING AVERAGE

Donna's Garden Supply (see Example 1) wants to forecast storage shed sales by weighting the past 3 months, with more weight given to recent data to make them more significant.

APPROACH ▶ Assign more weight to recent data, as follows:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

SOLUTION ▶ The results of this weighted-average forecast are as follows:

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\frac{1}{3}$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\frac{5}{6}$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\frac{1}{2}$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\frac{1}{3}$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\frac{1}{3}$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\frac{2}{3}$

The forecast for January is $15\frac{1}{3}$. Do you see how this number is computed?

INSIGHT ▶ In this particular forecasting situation, you can see that more heavily weighting the latest month provides a more accurate projection.

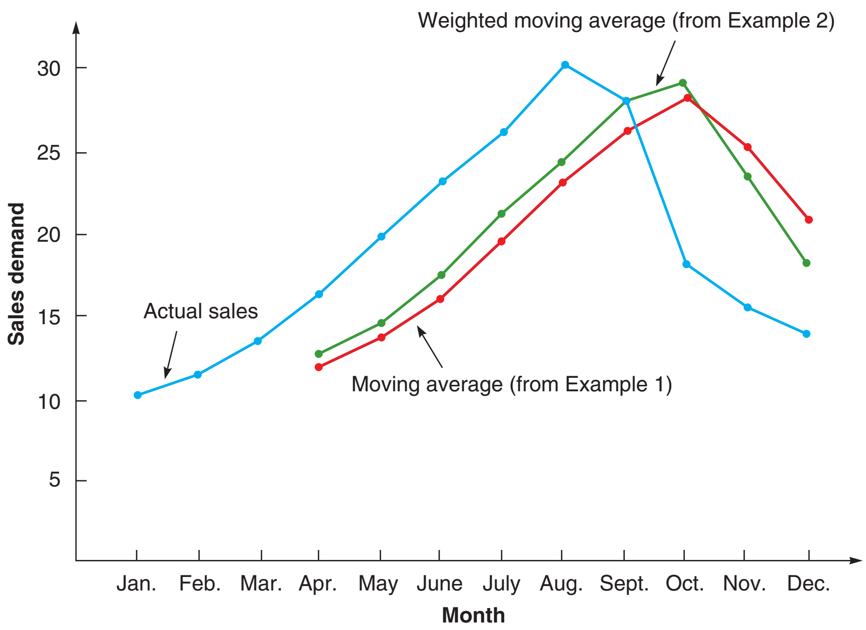
LEARNING EXERCISE ▶ If the assigned weights were 0.50, 0.33, and 0.17 (instead of 3, 2, and 1), what is the forecast for January's weighted moving average? Why? [Answer: There is no change. These are the same *relative* weights. Note that $\sum$ weights = 1 now, so there is no need for a denominator. When the weights sum to 1, calculations tend to be simpler.]

RELATED PROBLEMS ▶ 4.1b, 4.2c, 4.5c, 4.6, 4.7, 4.10b (4.38 is available in MyOMLab)

EXCEL OM Data File **Ch04Ex2.xls** can be found in MyOMLab.

Figure 4.2

Actual Demand vs. Moving-Average and Weighted-Moving-Average Methods for Donna's Garden Supply



STUDENT TIP

Moving-average methods always lag behind when there is a trend present, as shown by the blue line (actual sales) for January through August.

Both simple and weighted moving averages are effective in smoothing out sudden fluctuations in the demand pattern to provide stable estimates. Moving averages do, however, present three problems:

1. Increasing the size of n (the number of periods averaged) does smooth out fluctuations better, but it makes the method less sensitive to changes in the data.
2. Moving averages cannot pick up trends very well. Because they are averages, they will always stay within past levels and will not predict changes to either higher or lower levels. That is, they *lag* the actual values.
3. Moving averages require extensive records of past data.

Figure 4.2, a plot of the data in Examples 1 and 2, illustrates the lag effect of the moving-average models. Note that both the moving-average and weighted-moving-average lines lag the actual demand. The weighted moving average, however, usually reacts more quickly to demand changes. Even in periods of downturn (see November and December), it more closely tracks the demand.

Exponential Smoothing

Exponential smoothing

A weighted-moving-average forecasting technique in which data points are weighted by an exponential function.

Exponential smoothing is another weighted-moving-average forecasting method. It involves very *little* record keeping of past data and is fairly easy to use. The basic exponential smoothing formula can be shown as follows:

$$\text{New forecast} = \text{Last period's forecast} + \alpha\,(\text{Last period's actual demand} - \text{Last period's forecast}) \quad (4\text{-}3)$$

Smoothing constant

The weighting factor used in an exponential smoothing forecast, a number greater than or equal to 0 and less than or equal to 1.

where α is a weight, or **smoothing constant**, chosen by the forecaster, that has a value greater than or equal to 0 and less than or equal to 1. Equation (4-3) can also be written mathematically as:

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (4\text{-}4)$$

where
F_t = new forecast
F_{t-1} = previous period's forecast
α = smoothing (or weighting) constant ($0 \le \alpha \le 1$)
A_{t-1} = previous period's actual demand

The concept is not complex. The latest estimate of demand is equal to the old forecast adjusted by a fraction of the difference between the last period's actual demand and last period's forecast. Example 3 shows how to use exponential smoothing to derive a forecast.

Example 3

DETERMINING A FORECAST VIA EXPONENTIAL SMOOTHING

In January, a car dealer predicted February demand for 142 Ford Mustangs. Actual February demand was 153 autos. Using a smoothing constant chosen by management of $\alpha = .20$, the dealer wants to forecast March demand using the exponential smoothing model.

APPROACH ▶ The exponential smoothing model in Equations (4-3) and (4-4) can be applied.

SOLUTION ▶ Substituting the sample data into the formula, we obtain:

$$\begin{aligned}\text{New forecast (for March demand)} &= 142 + .2(153 - 142) = 142 + 2.2\\ &= 144.2\end{aligned}$$

Thus, the March demand forecast for Ford Mustangs is rounded to 144.

INSIGHT ▶ Using just two pieces of data, the forecast and the actual demand, plus a smoothing constant, we developed a forecast of 144 Ford Mustangs for March.

LEARNING EXERCISE ▶ If the smoothing constant is changed to .30, what is the new forecast? [Answer: 145.3]

RELATED PROBLEMS ▶ 4.1c, 4.3, 4.4, 4.5d, 4.6, 4.9d, 4.11, 4.12, 4.13a, 4.17, 4.18, 4.31, 4.33, 4.34 (4.36, 4.61a are available in MyOMLab)

The *smoothing constant*, α, is generally in the range from .05 to .50 for business applications. It can be changed to give more weight to recent data (when α is high) or more weight to past data (when α is low). When α reaches the extreme of 1.0, then in Equation (4-4), $F_t = 1.0A_{t-1}$. All the older values drop out, and the forecast becomes identical to the naive model mentioned earlier in this chapter. That is, the forecast for the next period is just the same as this period's demand.

The following table helps illustrate this concept. For example, when $\alpha = .5$, we can see that the new forecast is based almost entirely on demand in the last three or four periods. When $\alpha = .1$, the forecast places little weight on recent demand and takes many periods (about 19) of historical values into account.

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4TH MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5TH MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Selecting the Smoothing Constant Exponential smoothing has been successfully applied in virtually every type of business. However, the appropriate value of the smoothing constant, α, can make the difference between an accurate forecast and an inaccurate forecast. High values of α are chosen when the underlying average is likely to change. Low values of α are used when the underlying average is fairly stable. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.

STUDENT TIP
Forecasts tend to be more accurate as they become shorter. Therefore, forecast error also tends to drop with shorter forecasts.

Measuring Forecast Error

The overall accuracy of any forecasting model—moving average, exponential smoothing, or other—can be determined by comparing the forecasted values with the actual or observed values. If F_t denotes the forecast in period t, and A_t denotes the actual demand in period t, the *forecast error* (or deviation) is defined as:

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value}$$
$$= A_t - F_t$$

LO 4.4 *Compute* three measures of forecast accuracy

Several measures are used in practice to calculate the overall forecast error. These measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE). We now describe and give an example of each.

Mean absolute deviation (MAD)
A measure of the overall forecast error for a model.

Mean Absolute Deviation The first measure of the overall forecast error for a model is the **mean absolute deviation (MAD)**. This value is computed by taking the sum of the absolute values of the individual forecast errors (deviations) and dividing by the number of periods of data (n):

$$\text{MAD} = \frac{\sum|\text{Actual} - \text{Forecast}|}{n} \tag{4-5}$$

Example 4 applies MAD, as a measure of overall forecast error, by testing two values of α.

Example 4

DETERMINING THE MEAN ABSOLUTE DEVIATION (MAD)

During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain from ships. The port's operations manager wants to test the use of exponential smoothing to see how well the technique works in predicting tonnage unloaded. He guesses that the forecast of grain unloaded in the first quarter was 175 tons. Two values of α are to be examined: $\alpha = .10$ and $\alpha = .50$.

APPROACH ▶ Compare the actual data with the data we forecast (using each of the two α values) and then find the absolute deviation and MADs.

SOLUTION ▶ The following table shows the *detailed* calculations for $\alpha = .10$ only:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	FORECAST WITH $\alpha = .50$
1	180	175	175
2	168	175.50 = 175.00 + .10(180 − 175)	177.50
3	159	174.75 = 175.50 + .10(168 − 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 − 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 − 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 − 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 − 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 − 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 − 178.22)	184.15

To evaluate the accuracy of each smoothing constant, we can compute forecast errors in terms of absolute deviations and MADs:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	ABSOLUTE DEVIATION FOR $\alpha = .10$	FORECAST WITH $\alpha = .50$	ABSOLUTE DEVIATION FOR $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
Sum of absolute deviations:			82.45		98.62
$\text{MAD} = \frac{\sum\|\text{Deviations}\|}{n}$			10.31		12.33

INSIGHT ▶ On the basis of this comparison of the two MADs, a smoothing constant of $\alpha = .10$ is preferred to $\alpha = .50$ because its MAD is smaller.

LEARNING EXERCISE ▶ If the smoothing constant is changed from $\alpha = .10$ to $\alpha = .20$, what is the new MAD? [Answer: 10.21.]

RELATED PROBLEMS ▶ 4.5b, 4.8c, 4.9c, 4.14, 4.23, 4.59b (4.35d, 4.37a, 4.38c, 4.61b are available in MyOMLab)

EXCEL OM Data File **Ch04Ex4a.xls** and **Ch04Ex4b.xls** can be found in MyOMLab.

ACTIVE MODEL 4.2 This example is further illustrated in Active Model 4.2 in MyOMLab.

Most computerized forecasting software includes a feature that automatically finds the smoothing constant with the lowest forecast error. Some software modifies the α value if errors become larger than acceptable.

Mean Squared Error The **mean squared error (MSE)** is a second way of measuring overall forecast error. MSE is the average of the squared differences between the forecasted and observed values. Its formula is:

Mean squared error (MSE)
The average of the squared differences between the forecasted and observed values.

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} \tag{4-6}$$

Example 5 finds the MSE for the Port of Baltimore problem introduced in Example 4.

Example 5

DETERMINING THE MEAN SQUARED ERROR (MSE)

The operations manager for the Port of Baltimore now wants to compute MSE for $\alpha = .10$.

APPROACH ▶ Using the same forecast data for $\alpha = .10$ from Example 4, compute the MSE with Equation (4-6).

SOLUTION ▶

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	$(\text{ERROR})^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$
3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum of errors squared = 1,526.52

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} = 1{,}526.52/8 = 190.8$$

INSIGHT ▶ Is this MSE = 190.8 good or bad? It all depends on the MSEs for other forecasting approaches. A low MSE is better because we want to minimize MSE. MSE exaggerates errors because it squares them.

LEARNING EXERCISE ▶ Find the MSE for $\alpha = .50$. [Answer: MSE = 195.24. The result indicates that $\alpha = .10$ is a better choice because we seek a lower MSE. Coincidentally, this is the same conclusion we reached using MAD in Example 4.]

RELATED PROBLEMS ▶ 4.8d, 4.11c, 4.14, 4.15c, 4.16c, 4.20 (4.35d, 4.37b are available in MyOMLab)

The MSE tends to accentuate large deviations due to the squared term. For example, if the forecast error for period 1 is twice as large as the error for period 2, the squared error in period 1 is four times as large as that for period 2. Hence, using MSE as the measure of forecast error typically indicates that we prefer to have several smaller deviations rather than even one large deviation.

Mean absolute percent error (MAPE)
The average of the absolute differences between the forecast and actual values, expressed as a percent of actual values.

Mean Absolute Percent Error A problem with both the MAD and MSE is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousands, the MAD and MSE values can be very large. To avoid this problem, we can use the **mean absolute percent error (MAPE)**. This is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. That is, if we have forecasted and actual values for n periods, the MAPE is calculated as:

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100\,|\text{Actual}_i - \text{Forecast}_i|\,/\,\text{Actual}_i}{n} \tag{4-7}$$

Example 6 illustrates the calculations using the data from Examples 4 and 5.

Example 6

DETERMINING THE MEAN ABSOLUTE PERCENT ERROR (MAPE)

The Port of Baltimore wants to now calculate the MAPE when $\alpha = .10$.

APPROACH ▶ Equation (4-7) is applied to the forecast data computed in Example 4.

SOLUTION ▶

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR 100 (|ERROR|/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

INSIGHT ▶ MAPE expresses the error as a percent of the actual values, undistorted by a single large value.

LEARNING EXERCISE ▶ What is MAPE when α is .50? [Answer: MAPE = 6.75%. As was the case with MAD and MSE, the $\alpha = .1$ was preferable for this series of data.]

RELATED PROBLEMS ▶ 4.8e, 4.29c

The MAPE is perhaps the easiest measure to interpret. For example, a result that the MAPE is 6% is a clear statement that is not dependent on issues such as the magnitude of the input data.

Table 4.1 summarizes how MAD, MSE, and MAPE differ.

Exponential Smoothing with Trend Adjustment

Simple exponential smoothing, the technique we just illustrated in Examples 3 to 6, is like any other moving-average technique: It fails to respond to trends. Other forecasting techniques that can deal with trends are certainly available. However, because exponential smoothing is such a popular modeling approach in business, let us look at it in more detail.

TABLE 4.1 Comparison of Measures of Forecast Error

MEASURE	MEANING	EQUATION	APPLICATION TO CHAPTER EXAMPLE
Mean absolute deviation (MAD)	How much the forecast missed the target	$\text{MAD} = \dfrac{\sum\lvert\text{Actual} - \text{Forecast}\rvert}{n}$ (4-5)	For $\alpha = .10$ in Example 4, the forecast for grain unloaded was off by an average of 10.31 tons.
Mean squared error (MSE)	The square of how much the forecast missed the target	$\text{MSE} = \dfrac{\sum(\text{Forecast errors})^2}{n}$ (4-6)	For $\alpha = .10$ in Example 5, the square of the forecast error was 190.8. This number does not have a physical meaning but is useful when compared to the MSE of another forecast.
Mean absolute percent error (MAPE)	The average percent error	$\text{MAPE} = \dfrac{\sum_{i=1}^{n} 100\lvert\text{Actual}_i - \text{Forecast}_i\rvert/\text{Actual}_i}{n}$ (4-7)	For $\alpha = .10$ in Example 6, the forecast is off by 5.59% on average. As in Examples 4 and 5, some forecasts were too high, and some were low.

Here is why exponential smoothing must be modified when a trend is present. Assume that demand for our product or service has been increasing by 100 units per month and that we have been forecasting with $\alpha = 0.4$ in our exponential smoothing model. The following table shows a severe lag in the second, third, fourth, and fifth months, even when our initial estimate for month 1 is perfect:

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1–5
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\text{Forecast including trend } (FIT_t) = \text{Exponentially smoothed forecast average } (F_t) + \text{Exponentially smoothed trend } (T_t) \quad (4\text{-}8)$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \quad (4\text{-}9)$$

$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$

or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4\text{-}10)$$

where F_t = exponentially smoothed forecast average of the data series in period t
T_t = exponentially smoothed trend in period t
A_t = actual demand in period t
α = smoothing constant for the average ($0 \le \alpha \le 1$)
β = smoothing constant for the trend ($0 \le \beta \le 1$)

So the three steps to compute a trend-adjusted forecast are:

STEP 1: Compute F_t, the exponentially smoothed forecast average for period t, using Equation (4-9).

STEP 2: Compute the smoothed trend, T_t, using Equation (4-10).

STEP 3: Calculate the forecast including trend, FIT_t, by the formula $FIT_t = F_t + T_t$ [from Equation (4-8)].

Example 7 shows how to use trend-adjusted exponential smoothing.

Example 7

COMPUTING A TREND-ADJUSTED EXPONENTIAL SMOOTHING FORECAST

A large Portland manufacturer wants to forecast demand for a piece of pollution-control equipment. A review of past sales, as shown below, indicates that an increasing trend is present:

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

Smoothing constants are assigned the values of $\alpha = .2$ and $\beta = .4$. The firm assumes the initial forecast average for month 1 (F_1) was 11 units and the trend over that period (T_1) was 2 units.

APPROACH ▶ A trend-adjusted exponential smoothing model, using Equations (4-9), (4-10), and (4-8) and the three steps above, is employed.

SOLUTION ▶

Step 1: Forecast average for month 2:

$$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$$
$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$
$$= 2.4 + (.8)(13) = 2.4 + 10.4 = 12.8 \text{ units}$$

Step 2: Compute the trend in period 2:

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$
$$= .4(12.8 - 11) + (1 - .4)(2)$$
$$= (.4)(1.8) + (.6)(2) = .72 + 1.2 = 1.92$$

Step 3: Compute the forecast including trend (FIT_t):

$$FIT_2 = F_2 + T_2$$
$$= 12.8 + 1.92$$
$$= 14.72 \text{ units}$$

We will also do the same calculations for the third month:

Step 1: $F_3 = \alpha A_2 + (1 - \alpha)(F_2 + T_2) = (.2)(17) + (1 - .2)(12.8 + 1.92)$
$= 3.4 + (.8)(14.72) = 3.4 + 11.78 = 15.18$

Step 2: $T_3 = \beta(F_3 - F_2) + (1 - \beta)T_2 = (.4)(15.18 - 12.8) + (1 - .4)(1.92)$
$= (.4)(2.38) + (.6)(1.92) = .952 + 1.152 = 2.10$

Step 3: $FIT_3 = F_3 + T_3$
$= 15.18 + 2.10 = 17.28.$

Table 4.2 completes the forecasts for the 10-month period.

TABLE 4.2 Forecast with α = .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10	—	32.48	2.68	35.16

INSIGHT ▶ Figure 4.3 compares actual demand (A_t) to an exponential smoothing forecast that includes trend (FIT_t). *FIT* picks up the trend in actual demand. A simple exponential smoothing model (as we saw in Examples 3 and 4) trails far behind.

LEARNING EXERCISE ▶ Using the data for actual demand for the 9 months, compute the exponentially smoothed forecast average *without* trend [using Equation (4-4) as we did earlier in Examples 3 and 4]. Apply α = .2, and assume an initial forecast average for month 1 of 11 units. Then plot the months 2–10 forecast values on Figure 4.3. What do you notice? [Answer: Month 10 forecast = 24.65. All the points are below and lag the trend-adjusted forecast.]

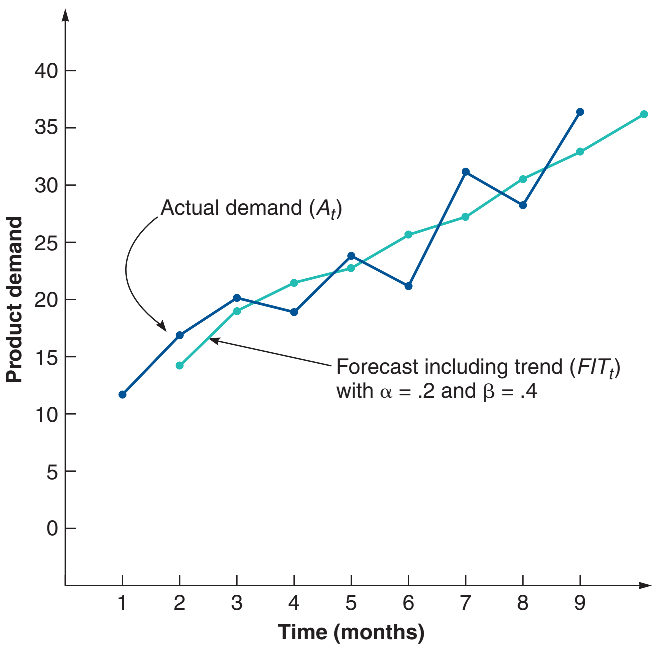
RELATED PROBLEMS ▶ 4.19, 4.20, 4.21, 4.22, 4.32

ACTIVE MODEL 4.3 This example is further illustrated in Active Model 4.3 in MyOMLab.

EXCEL OM Data File **Ch04Ex7.xis** can be found in MyOMLab.

Figure **4.3**

Exponential Smoothing with Trend-Adjustment Forecasts Compared to Actual Demand Data



The value of the trend-smoothing constant, β, resembles the α constant because a high β is more responsive to recent changes in trend. A low β gives less weight to the most recent trends and tends to smooth out the present trend. Values of β can be found by the trial-and-error approach or by using sophisticated commercial forecasting software, with the MAD used as a measure of comparison.

Simple exponential smoothing is often referred to as *first-order smoothing*, and trend-adjusted smoothing is called *second-order smoothing* or *double smoothing*. Other advanced exponential-smoothing models are also used, including seasonal-adjusted and triple smoothing.

Trend Projections

Trend projection

A time-series forecasting method that fits a trend line to a series of historical data points and then projects the line into the future for forecasts.

The last time-series forecasting method we will discuss is **trend projection**. This technique fits a trend line to a series of historical data points and then projects the slope of the line into the future for medium- to long-range forecasts. Several mathematical trend equations can be developed (for example, exponential and quadratic), but in this section, we will look at *linear* (straight-line) trends only.

If we decide to develop a linear trend line by a precise statistical method, we can apply the *least-squares method*. This approach results in a straight line that minimizes the sum of the squares of the vertical differences or deviations from the line to each of the actual observations. Figure 4.4 illustrates the least-squares approach.

A least-squares line is described in terms of its y-intercept (the height at which it intercepts the y-axis) and its expected change (slope). If we can compute the y-intercept and slope, we can express the line with the following equation:

$$\hat{y} = a + bx \tag{4-11}$$

where $\hat{y}$ (called "y hat") = computed value of the variable to be predicted (called the *dependent variable*)

a = y-axis intercept

b = slope of the regression line (or the rate of change in y for given changes in x)

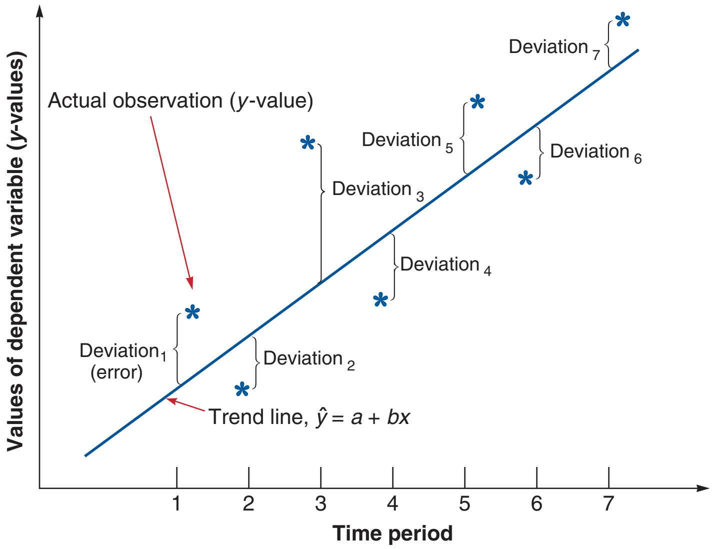
x = the independent variable (which in this case is *time*)

Statisticians have developed equations that we can use to find the values of a and b for any regression line. The slope b is found by:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} \tag{4-12}$$

Figure **4.4**

The Least-Squares Method for Finding the Best-Fitting Straight Line, Where the Asterisks Are the Locations of the Seven Actual Observations or Data Points



where b = slope of the regression line
Σ = summation sign
x = known values of the independent variable
y = known values of the dependent variable
$\bar{x}$ = average of the x-values
$\bar{y}$ = average of the y-values
n = number of data points or observations

We can compute the y-intercept a as follows:

$$a = \bar{y} - b\bar{x} \qquad (4\text{-}13)$$

Example 8 shows how to apply these concepts.

Example 8

FORECASTING WITH LEAST SQUARES

The demand for electric power at N.Y. Edison over the past 7 years is shown in the following table, in megawatts. The firm wants to forecast next year's demand by fitting a straight-line trend to these data.

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

APPROACH ▶ Equations (4-12) and (4-13) can be used to create the trend projection model.

SOLUTION ▶

YEAR (x)	ELECTRIC POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\Sigma y}{n} = \frac{692}{7} = 98.86$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

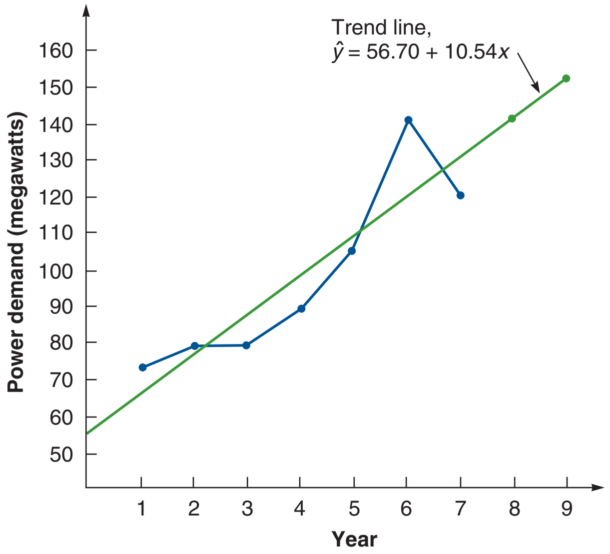
Thus, the least-squares trend equation is $\hat{y} = 56.70 + 10.54x$. To project demand next year, $x = 8$:

$$\begin{aligned}\text{Demand in year 8} &= 56.70 + 10.54(8)\\ &= 141.02, \text{ or } 141 \text{ megawatts}\end{aligned}$$

INSIGHT ▶ To evaluate the model, we plot both the historical demand and the trend line in Figure 4.5. In this case, we may wish to be cautious and try to understand the year 6 to year 7 swing in demand.

Figure 4.5

Electrical Power and the Computed Trend Line



LEARNING EXERCISE ▶ Estimate demand for year 9. [Answer: 151.56, or 152 megawatts.]

RELATED PROBLEMS ▶ 4.6, 4.13c, 4.16, 4.24, 4.30, 4.34 (4.39, 4.42 are available in MyOMLab)

EXCEL OM Data File **Ch04Ex8.xls** can be found in MyOMLab.

ACTIVE MODEL 4.4 This example is further illustrated in Active Model 4.4 in MyOMLab.

Notes on the Use of the Least-Squares Method Using the least-squares method implies that we have met three requirements:

1. We always plot the data because least-squares data assume a linear relationship. If a curve appears to be present, curvilinear analysis is probably needed.
2. We do not predict time periods far beyond our given database. For example, if we have 20 months' worth of average prices of Microsoft stock, we can forecast only 3 or 4 months into the future. Forecasts beyond that have little statistical validity. Thus, you cannot take 5 years' worth of sales data and project 10 years into the future. The world is too uncertain.
3. Deviations around the least-squares line (see Figure 4.4) are assumed to be random and normally distributed, with most observations close to the line and only a smaller number farther out.

Seasonal Variations in Data

Seasonal variations
Regular upward or downward movements in a time series that tie to recurring events.

Seasonal variations in data are regular movements in a time series that relate to recurring events such as weather or holidays. Demand for coal and fuel oil, for example, peaks during cold winter months. Demand for golf clubs or sunscreen may be highest in summer.

Seasonality may be applied to hourly, daily, weekly, monthly, or other recurring patterns. Fast-food restaurants experience *daily* surges at noon and again at 5 P.M. Movie theaters see higher demand on Friday and Saturday evenings. The post office, Toys "Я" Us, The Christmas Store, and Hallmark Card Shops also exhibit seasonal variation in customer traffic and sales.

STUDENT TIP
John Deere understands seasonal variations: It has been able to obtain 70% of its orders in advance of seasonal use so it can smooth production.

Similarly, understanding seasonal variations is important for capacity planning in organizations that handle peak loads. These include electric power companies during extreme cold and warm periods, banks on Friday afternoons, and buses and subways during the morning and evening rush hours.


CityFiles/Getty Images


Dick Loek/Getty Images

Demand for many products is seasonal. Yamaha, the manufacturer of this jet ski and snowmobile, produces products with complementary demands to address seasonal fluctuations.

Time-series forecasts like those in Example 8 involve reviewing the trend of data over a series of time periods. The presence of seasonality makes adjustments in trend-line forecasts necessary. Seasonality is expressed in terms of the amount that actual values differ from average values in the time series. Analyzing data in monthly or quarterly terms usually makes it easy for a statistician to spot seasonal patterns. Seasonal indices can then be developed by several common methods.

In what is called a *multiplicative seasonal model*, seasonal factors are multiplied by an estimate of average demand to produce a seasonal forecast. Our assumption in this section is that trend has been removed from the data. Otherwise, the magnitude of the seasonal data will be distorted by the trend.

Here are the steps we will follow for a company that has "seasons" of 1 month:

LO 4.5 *Develop* seasonal indices

1. Find the *average historical demand each season* (or month in this case) by summing the demand for that month in each year and dividing by the number of years of data available. For example, if, in January, we have seen sales of 8, 6, and 10 over the past 3 years, average January demand equals $(8 + 6 + 10)/3 = 8$ units.
2. Compute the *average demand over all months* by dividing the total average annual demand by the number of seasons. For example, if the total average demand for a year is 120 units and there are 12 seasons (each month), the average monthly demand is $120/12 = 10$ units.
3. Compute a *seasonal index* for each season by dividing that *month's* historical average demand (from Step 1) by the average demand over all months (from Step 2). For example, if the average historical January demand over the past 3 years is 8 units and the average demand over all months is 10 units, the seasonal index for January is $8/10 = .80$. Likewise, a seasonal index of 1.20 for February would mean that February's demand is 20% larger than the average demand over all months.
4. Estimate next year's total annual demand.
5. Divide this estimate of total annual demand by the number of seasons, then multiply it by the seasonal index for each month. This provides the *seasonal forecast*.

Example 9 illustrates this procedure as it computes seasonal indices from historical data.

Example 9

DETERMINING SEASONAL INDICES

A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past 3 years, by month, are available.

APPROACH ▶ Follow the five steps listed above.

SOLUTION ▶

	DEMAND					
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND[a]	SEASONAL INDEX[b]
Jan.	80	85	105	90	94	.957 (= 90/94)
Feb.	70	85	85	80	94	.851 (= 80/94)
Mar.	80	93	82	85	94	.904 (= 85/94)
Apr.	90	95	115	100	94	1.064 (= 100/94)
May	113	125	131	123	94	1.309 (= 123/94)
June	110	115	120	115	94	1.223 (= 115/94)
July	100	102	113	105	94	1.117 (= 105/94)
Aug.	88	102	110	100	94	1.064 (= 100/94)
Sept.	85	90	95	90	94	.957 (= 90/94)
Oct.	77	78	85	80	94	.851 (= 80/94)
Nov.	75	82	83	80	94	.851 (= 80/94)
Dec.	82	78	80	80	94	.851 (= 80/94)

Total average annual demand = 1,128

[a]Average monthly demand $= \frac{1{,}128}{12 \text{ months}} = 94.$ [b]Seasonal index $= \frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}.$

If we expect the annual demand for computers to be 1,200 units next year, we would use these seasonal indices to forecast the monthly demand as follows:

MONTH	DEMAND	MONTH	DEMAND
Jan.	$\frac{1{,}200}{12} \times .957 = 96$	July	$\frac{1{,}200}{12} \times 1.117 = 112$
Feb.	$\frac{1{,}200}{12} \times .851 = 85$	Aug.	$\frac{1{,}200}{12} \times 1.064 = 106$
Mar.	$\frac{1{,}200}{12} \times .904 = 90$	Sept.	$\frac{1{,}200}{12} \times .957 = 96$
Apr.	$\frac{1{,}200}{12} \times 1.064 = 106$	Oct.	$\frac{1{,}200}{12} \times .851 = 85$
May	$\frac{1{,}200}{12} \times 1.309 = 131$	Nov.	$\frac{1{,}200}{12} \times .851 = 85$
June	$\frac{1{,}200}{12} \times 1.223 = 122$	Dec.	$\frac{1{,}200}{12} \times .851 = 85$

INSIGHT ▶ Think of these indices as percentages of average sales. The average sales (without seasonality) would be 94, but with seasonality, sales fluctuate from 85% to 131% of average.

LEARNING EXERCISE ▶ If next year's annual demand is 1,150 laptops (instead of 1,200), what will the January, February, and March forecasts be? [Answer: 91.7, 81.5, and 86.6, which can be rounded to 92, 82, and 87.]

RELATED PROBLEMS ▶ 4.26, 4.27 (4.40, 4.41a are available in MyOMLab)

EXCEL OM Data File **Ch04Ex9.xls** can be found in MyOMLab.

For simplicity, only 3 periods (years) are used for each monthly index in the preceding example. Example 10 illustrates how indices that have already been prepared can be applied to adjust trend-line forecasts for seasonality.

Example 10

APPLYING BOTH TREND AND SEASONAL INDICES

San Diego Hospital wants to improve its forecasting by applying both trend and seasonal indices to 66 months of data it has collected. It will then forecast "patient-days" over the coming year.

APPROACH ▶ A trend line is created; then monthly seasonal indices are computed. Finally, a multiplicative seasonal model is used to forecast months 67 to 78.

SOLUTION ▶ Using 66 months of adult inpatient hospital days, the following equation was computed:

$$\hat{y} = 8{,}090 + 21.5x$$

where

$$\hat{y} = \text{patient days}$$
$$x = \text{time, in months}$$

Based on this model, which reflects only trend data, the hospital forecasts patient days for the next month (period 67) to be:

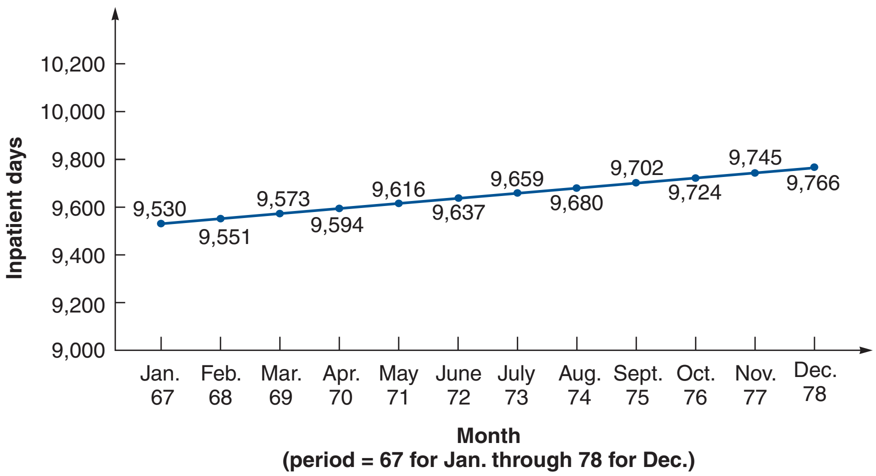
$$\text{Patient days} = 8{,}090 + (21.5)(67) = 9{,}530 \text{ (trend only)}$$

While this model, as plotted in Figure 4.6, recognized the upward trend line in the demand for inpatient services, it ignored the seasonality that the administration knew to be present.

Figure 4.6

Trend Data for San Diego Hospital

Source: From "Modern Methods Improve Hospital Forecasting" by W. E. Sterk and E. G. Shryock from *Healthcare Financial Management* 41, no. 3, p. 97. Reprinted by permission of Healthcare Financial Management Association.



The following table provides seasonal indices based on the same 66 months. Such seasonal data, by the way, were found to be typical of hospitals nationwide.

Seasonality Indices for Adult Inpatient Days at San Diego Hospital

MONTH	SEASONALITY INDEX	MONTH	SEASONALITY INDEX
January	1.04	July	1.03
February	0.97	August	1.04
March	1.02	September	0.97
April	1.01	October	1.00
May	0.99	November	0.96
June	0.99	December	0.98

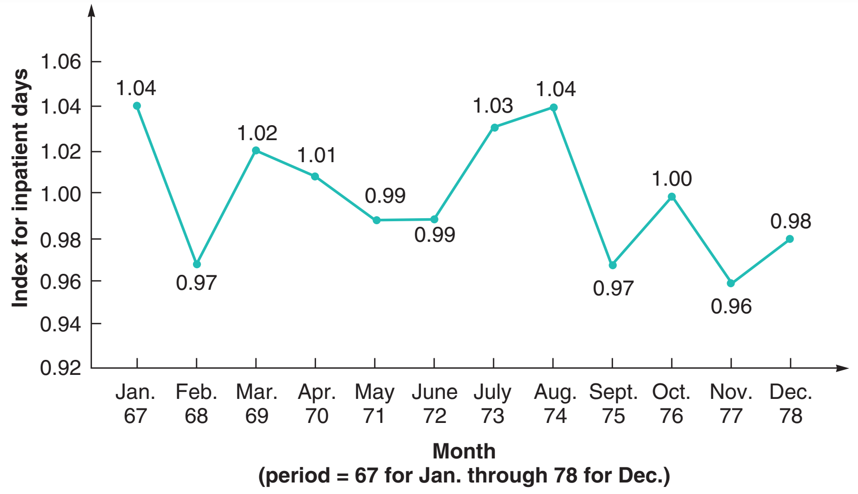
These seasonal indices are graphed in Figure 4.7. Note that January, March, July, and August seem to exhibit significantly higher patient days on average, while February, September, November, and December experience lower patient days.

However, neither the trend data nor the seasonal data alone provide a reasonable forecast for the hospital. Only when the hospital multiplied the trend-adjusted data by the appropriate seasonal index did it obtain good forecasts. Thus, for period 67 (January):

$$\text{Patient days} = (\text{Trend-adjusted forecast})(\text{Monthly seasonal index}) = (9{,}530)(1.04) = 9{,}911$$

Figure **4.7**

Seasonal Index for San Diego Hospital



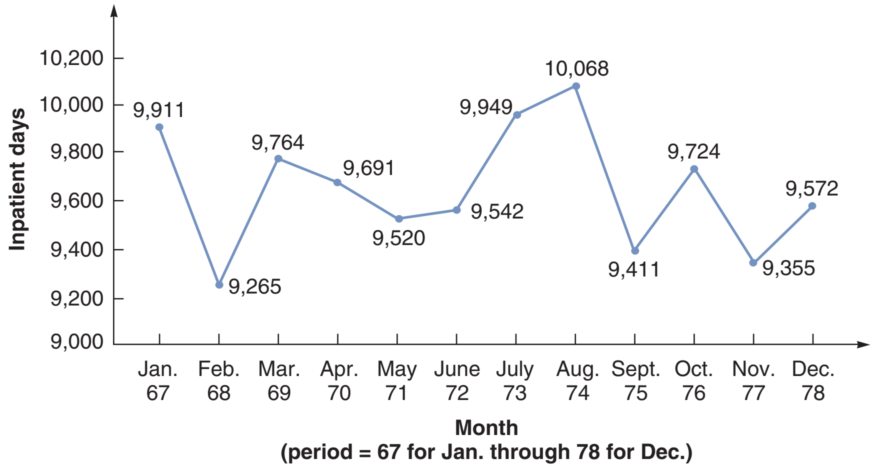
The patient-days for each month are:

Period	67	68	69	70	71	72	73	74	75	76	77	78
Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Forecast with Trend & Seasonality	9,911	9,265	9,764	9,691	9,520	9,542	9,949	10,068	9,411	9,724	9,355	9,572

A graph showing the forecast that combines both trend and seasonality appears in Figure 4.8.

Figure **4.8**

Combined Trend and Seasonal Forecast



INSIGHT ▶ Notice that with trend only, the September forecast is 9,702, but with both trend and seasonal adjustments, the forecast is 9,411. By combining trend and seasonal data, the hospital was better able to forecast inpatient days and the related staffing and budgeting vital to effective operations.

LEARNING EXERCISE ▶ If the slope of the trend line for patient-days is 22.0 (rather than 21.5) and the index for December is .99 (instead of .98), what is the new forecast for December inpatient days? [Answer: 9,708.]

RELATED PROBLEMS ▶ 4.25, 4.28

Example 11 further illustrates seasonality for quarterly data at a wholesaler.

Example 11

ADJUSTING TREND DATA WITH SEASONAL INDICES

Management at Jagoda Wholesalers, in Calgary, Canada, has used time-series regression based on point-of-sale data to forecast sales for the next 4 quarters. Sales estimates are \$100,000, \$120,000, \$140,000, and \$160,000 for the respective quarters. Seasonal indices for the four quarters have been found to be 1.30, .90, .70, and 1.10, respectively.

APPROACH ▶ To compute a seasonalized or adjusted sales forecast, we just multiply each seasonal index by the appropriate trend forecast:

$$\hat{y}_{\text{seasonal}} = \text{Index} \times \hat{y}_{\text{trend forecast}}$$

SOLUTION ▶

$$\begin{aligned}
&\text{Quarter I:} && \hat{y}_{\text{I}} = (1.30)(\$100{,}000) = \$130{,}000\\
&\text{Quarter II:} && \hat{y}_{\text{II}} = (.90)(\$120{,}000) = \$108{,}000\\
&\text{Quarter III:} && \hat{y}_{\text{III}} = (.70)(\$140{,}000) = \$98{,}000\\
&\text{Quarter IV:} && \hat{y}_{\text{IV}} = (1.10)(\$160{,}000) = \$176{,}000
\end{aligned}$$

INSIGHT ▶ The straight-line trend forecast is now adjusted to reflect the seasonal changes.

LEARNING EXERCISE ▶ If the sales forecast for Quarter IV was \$180,000 (rather than \$160,000), what would be the seasonally adjusted forecast? [Answer: \$198,000.]

RELATED PROBLEMS ▶ 4.25, 4.28 (4.41b is available in MyOMLab)

Cyclical Variations in Data

Cycles are like seasonal variations in data but occur every several *years*, not weeks, months, or quarters. Forecasting cyclical variations in a time series is difficult. This is because cycles include a wide variety of factors that cause the economy to go from recession to expansion to recession over a period of years. These factors include national or industrywide overexpansion in times of euphoria and contraction in times of concern. Forecasting demand for individual products can also be driven by product life cycles—the stages products go through from introduction through decline. Life cycles exist for virtually all products; striking examples include floppy disks, video recorders, and the original Game Boy. We leave cyclical analysis to forecasting texts.

Developing associative techniques of variables that affect one another is our next topic.

Cycles
Patterns in the data that occur every several years.

Associative Forecasting Methods: Regression and Correlation Analysis

STUDENT TIP
We now deal with the same mathematical model that we saw earlier, the least-squares method. But we use any potential "cause-and-effect" variable as *x*.

Unlike time-series forecasting, *associative forecasting* models usually consider *several* variables that are related to the quantity being predicted. Once these related variables have been found, a statistical model is built and used to forecast the item of interest. This approach is more powerful than the time-series methods that use only the historical values for the forecast variable.

Many factors can be considered in an associative analysis. For example, the sales of Dell PCs may be related to Dell's advertising budget, the company's prices, competitors' prices and promotional strategies, and even the nation's economy and unemployment rates. In this case, PC sales would be called the *dependent variable*, and the other variables would be called *independent variables*. The manager's job is to develop *the best statistical relationship between PC sales and the independent variables*. The most common quantitative associative forecasting model is **linear-regression analysis**.

Linear-regression analysis
A straight-line mathematical model to describe the functional relationships between independent and dependent variables.

Using Regression Analysis for Forecasting

We can use the same mathematical model that we employed in the least-squares method of trend projection to perform a linear-regression analysis. The dependent variables that we want to forecast will still be $\hat{y}$. But now the independent variable, x, need no longer be time. We use the equation:

$$\hat{y} = a + bx$$

where
$\hat{y}$ = value of the dependent variable (in our example, sales)
a = y-axis intercept
b = slope of the regression line
x = independent variable

LO 4.6 *Conduct* a regression and correlation analysis

Example 12 shows how to use linear regression.

Example 12

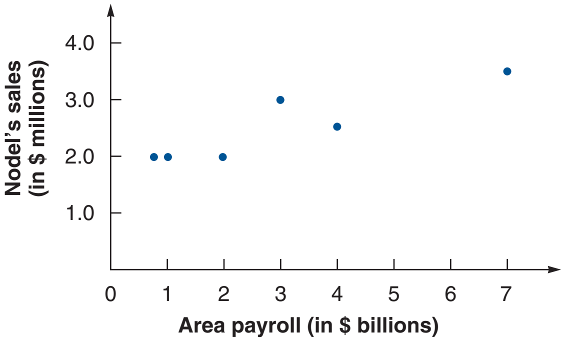
COMPUTING A LINEAR REGRESSION EQUATION

Nodel Construction Company renovates old homes in West Bloomfield, Michigan. Over time, the company has found that its dollar volume of renovation work is dependent on the West Bloomfield area payroll. Management wants to establish a mathematical relationship to help predict sales.

APPROACH ▶ Nodel's VP of operations has prepared the following table, which lists company revenues and the amount of money earned by wage earners in West Bloomfield during the past 6 years:

NODEL'S SALES (IN \$ MILLIONS), y	AREA PAYROLL (IN \$ BILLIONS), x	NODEL'S SALES (IN \$ MILLIONS), y	AREA PAYROLL (IN \$ BILLIONS), x
2.0	1	2.0	2
3.0	3	2.0	1
2.5	4	3.5	7

The VP needs to determine whether there is a straight-line (linear) relationship between area payroll and sales. He plots the known data on a scatter diagram:



STUDENT TIP A scatter diagram is a powerful data analysis tool. It helps quickly size up the relationship between two variables.

From the six data points, there appears to be a slight positive relationship between the independent variable (payroll) and the dependent variable (sales): As payroll increases, Nodel's sales tend to be higher.

SOLUTION ▶ We can find a mathematical equation by using the least-squares regression approach:

VIDEO 4.1 Forecasting Ticket Revenue for Orlando Magic Basketball Games

SALES, y	PAYROLL, x	x^2	xy
2.0	1	1	2.0
3.0	3	9	9.0
2.5	4	16	10.0
2.0	2	4	4.0
2.0	1	1	2.0
3.5	7	49	24.5
$\Sigma y = 15.0$	$\Sigma x = 18$	$\Sigma x^2 = 80$	$\Sigma xy = 51.5$

$$\bar{x} = \frac{\Sigma x}{6} = \frac{18}{6} = 3$$

$$\bar{y} = \frac{\Sigma y}{6} = \frac{15}{6} = 2.5$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - (6)(3^2)} = .25$$

$$a = \bar{y} - b\bar{x} = 2.5 - (.25)(3) = 1.75$$

The estimated regression equation, therefore, is:

$$\hat{y} = 1.75 + .25x$$

or:

$$\text{Sales} = 1.75 + .25\,(\text{payroll})$$

If the local chamber of commerce predicts that the West Bloomfield area payroll will be \$6 billion next year, we can estimate sales for Nodel with the regression equation:

$$\text{Sales (in \$ millions)} = 1.75 + .25(6) \\ = 1.75 + 1.50 = 3.25$$

or:

$$\text{Sales} = \$3{,}250{,}000$$

INSIGHT ▶ Given our assumptions of a straight-line relationship between payroll and sales, we now have an indication of the slope of that relationship: on average, sales increase at the rate of $\frac{1}{4}$ million dollars for every billion dollars in the local area payroll. This is because $b = .25$.

LEARNING EXERCISE ▶ What are Nodel's sales when the local payroll is \$8 billion? [Answer: \$3.75 million.]

RELATED PROBLEMS ▶ 4.34, 4.43–4.48, 4.50–4.54 (4.56a, 4.57, 4.58 are available in MyOMLab)

EXCEL OM Data File **Ch04Ex12.xls** can be found in MyOMLab.

The final part of Example 12 shows a central weakness of associative forecasting methods like regression. Even when we have computed a regression equation, we must provide a forecast of the independent variable x—in this case, payroll—before estimating the dependent variable y for the next time period. Although this is not a problem for all forecasts, you can imagine the difficulty of determining future values of *some* common independent variables (e.g., unemployment rates, gross national product, price indices, and so on).

Standard Error of the Estimate

The forecast of \$3,250,000 for Nodel's sales in Example 12 is called a *point estimate* of y. The point estimate is really the *mean*, or *expected value*, of a distribution of possible values of sales. Figure 4.9 illustrates this concept.

To measure the accuracy of the regression estimates, we must compute the **standard error of the estimate**, $S_{y,x}$. This computation is called the *standard deviation of the regression:* It measures the error from the dependent variable, y, to the regression line, rather than to the mean. Equation (4-14) is a similar expression to that found in most statistics books for computing the standard deviation of an arithmetic mean:

Standard error of the estimate
A measure of variability around the regression line—its standard deviation.

$$S_{y,x} = \sqrt{\frac{\Sigma(y - y_c)^2}{n - 2}} \tag{4-14}$$

where y = y-value of each data point
y_c = computed value of the dependent variable, from the regression equation
n = number of data points

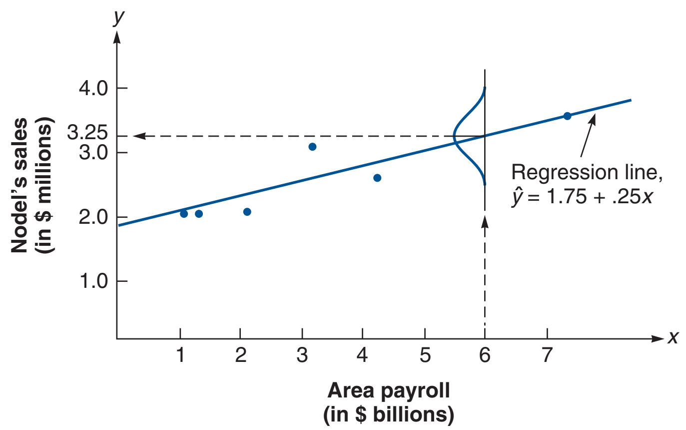
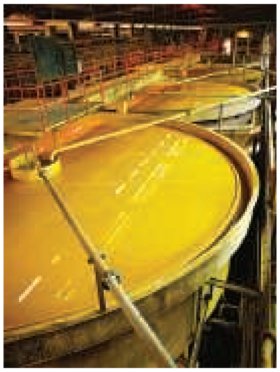


Figure **4.9**
Distribution about the Point Estimate of \$3.25 Million Sales



Michael Rosenfeld/Maximilian S/RGB Ventures/ SuperStock/Alamy

Glidden Paints' assembly lines require thousands of gallons every hour. To predict demand, the firm uses associative forecasting methods such as linear regression, with independent variables such as disposable personal income and GNP. Although housing starts would be a natural variable, Glidden found that it correlated poorly with past sales. It turns out that most Glidden paint is sold through retailers to customers who already own homes or businesses.

Equation (4-15) may look more complex, but it is actually an easier-to-use version of Equation (4-14). Both formulas provide the same answer and can be used in setting up prediction intervals around the point estimate:[2]

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n - 2}} \tag{4-15}$$

Example 13 shows how we would calculate the standard error of the estimate in Example 12.

Example 13

COMPUTING THE STANDARD ERROR OF THE ESTIMATE

Nodel's VP of operations now wants to know the error associated with the regression line computed in Example 12.

APPROACH ▶ Compute the standard error of the estimate, $S_{y,x}$, using Equation (4-15).

SOLUTION ▶ The only number we need that is not available to solve for $S_{y,x}$ is $\sum y^2$. Some quick addition reveals $\sum y^2 = 39.5$. Therefore:

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n - 2}}$$

$$= \sqrt{\frac{39.5 - 1.75(15.0) - .25(51.5)}{6 - 2}}$$

$$= \sqrt{.09375} = .306 \text{ (in \$ millions)}$$

The standard error of the estimate is then $306,000 in sales.

INSIGHT ▶ The interpretation of the standard error of the estimate is similar to the standard deviation; namely, ± 1 standard deviation = .6827. So there is a 68.27% chance of sales being $\pm$ $306,000 from the point estimate of $3,250,000.

LEARNING EXERCISE ▶ What is the probability sales will exceed $3,556,000? [Answer: About 16%.]

RELATED PROBLEMS ▶ 4.52e, 4.54b (4.56c, 4.57 are available in MyOMLab)

Correlation Coefficients for Regression Lines

Coefficient of correlation
A measure of the strength of the relationship between two variables.

The regression equation is one way of expressing the nature of the relationship between two variables. Regression lines are not "cause-and-effect" relationships. They merely describe the relationships among variables. The regression equation shows how one variable relates to the value and changes in another variable.

Another way to evaluate the relationship between two variables is to compute the **coefficient of correlation**. This measure expresses the degree or strength of the linear relationship (but note

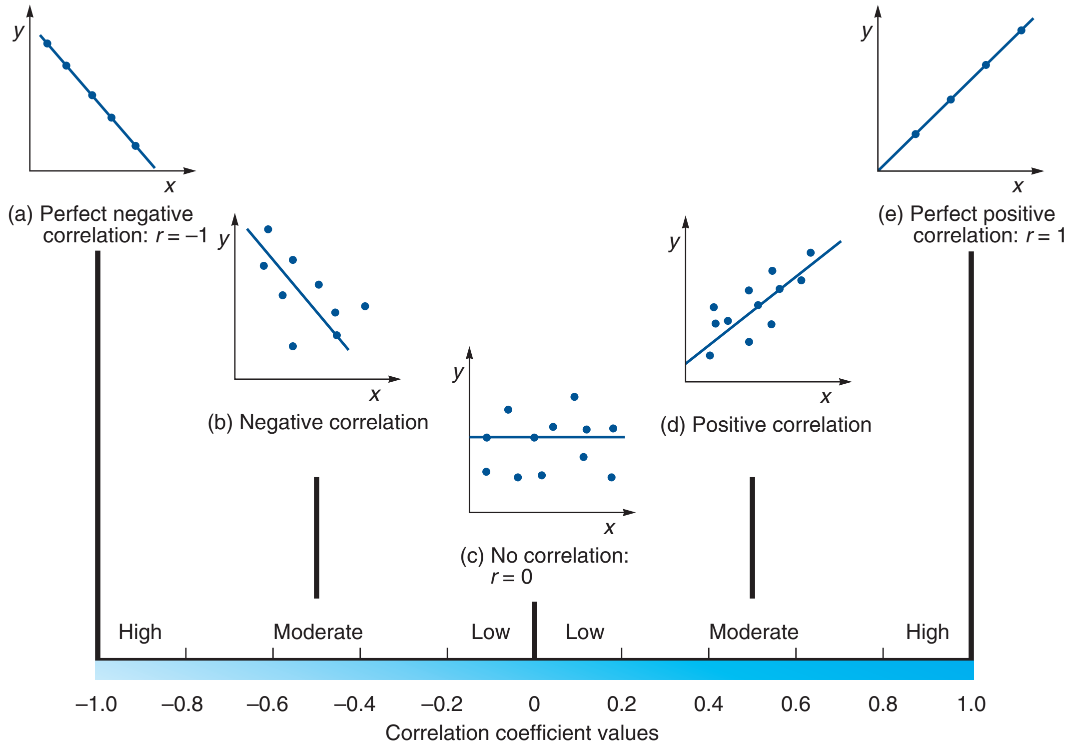


Figure 4.10

Five Values of the Correlation Coefficient

that correlation does not necessarily imply causality). Usually identified as r, the coefficient of correlation can be any number between $+1$ and -1. Figure 4.10 illustrates what different values of r might look like.

To compute r, we use much of the same data needed earlier to calculate a and b for the regression line. The rather lengthy equation for r is:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}} \qquad (4\text{-}16)$$

Example 14 shows how to calculate the coefficient of correlation for the data given in Examples 12 and 13.

Example 14

DETERMINING THE COEFFICIENT OF CORRELATION

In Example 12, we looked at the relationship between Nodel Construction Company's renovation sales and payroll in its hometown of West Bloomfield. The VP now wants to know the strength of the association between area payroll and sales.

APPROACH ▶ We compute the r value using Equation (4-16). We need to first add one more column of calculations—for y^2.

SOLUTION ▶ The data, including the column for y^2 and the calculations, are shown here:

y	x	x^2	xy	y^2
2.0	1	1	2.0	4.0
3.0	3	9	9.0	9.0
2.5	4	16	10.0	6.25
2.0	2	4	4.0	4.0
2.0	1	1	2.0	4.0
3.5	7	49	24.5	12.25
$\sum y = 15.0$	$\sum x = 18$	$\sum x^2 = 80$	$\sum xy = 51.5$	$\sum y^2 = 39.5$

$$r = \frac{(6)(51.5) - (18)(15.0)}{\sqrt{[(6)(80) - (18)^2][(6)(39.5) - (15.0)^2]}}$$
$$= \frac{309 - 270}{\sqrt{(156)(12)}} = \frac{39}{\sqrt{1,872}}$$
$$= \frac{39}{43.3} = .901$$

INSIGHT ▶ This r of .901 appears to be a significant correlation and helps confirm the closeness of the relationship between the two variables.

LEARNING EXERCISE ▶ If the coefficient of correlation was $-.901$ rather than $+.901$, what would this tell you? [Answer: The negative correlation would tell you that as payroll went up, Nodel's sales went down—a rather unlikely occurrence that would suggest you recheck your math.]

RELATED PROBLEMS ▶ 4.43d, 4.48d, 4.50c, 4.52f, 4.54b (4.56b, 4.57 are available in MyOMLab)

Coefficient of determination
A measure of the amount of variation in the dependent variable about its mean that is explained by the regression equation.

Although the coefficient of correlation is the measure most commonly used to describe the relationship between two variables, another measure does exist. It is called the **coefficient of determination** and is simply the square of the coefficient of correlation—namely, r^2. The value of r^2 will always be a positive number in the range $0 \le r^2 \le 1$. The coefficient of determination is the percent of variation in the dependent variable (y) that is explained by the regression equation. In Nodel's case, the value of r^2 is .81, indicating that 81% of the total variation is explained by the regression equation.

Multiple-Regression Analysis

Multiple regression
An associative forecasting method with more than one independent variable.

Multiple regression is a practical extension of the simple regression model we just explored. It allows us to build a model with several independent variables instead of just one variable. For example, if Nodel Construction wanted to include average annual interest rates in its model for forecasting renovation sales, the proper equation would be:

$$\hat{y} = a + b_1x_1 + b_2x_2 \tag{4-17}$$

where y = dependent variable, sales
a = a constant, the y intercept
x_1 and x_2 = values of the two independent variables, area payroll and interest rates, respectively
b_1 and b_2 = coefficients for the two independent variables

The mathematics of multiple regression becomes quite complex (and is usually tackled by computer), so we leave the formulas for a, b_1, and b_2 to statistics textbooks. However, Example 15 shows how to interpret Equation (4-17) in forecasting Nodel's sales.

Example 15

USING A MULTIPLE-REGRESSION EQUATION

Nodel Construction wants to see the impact of a second independent variable, interest rates, on its sales.

APPROACH ▶ The new multiple-regression line for Nodel Construction, calculated by computer software, is:

$$\hat{y} = 1.80 + .30x_1 - 5.0x_2$$

We also find that the new coefficient of correlation is .96, implying the inclusion of the variable x_2, interest rates, adds even more strength to the linear relationship.

SOLUTION ▶ We can now estimate Nodel's sales if we substitute values for next year's payroll and interest rate. If West Bloomfield's payroll will be $6 billion and the interest rate will be .12 (12%), sales will be forecast as:

$$\begin{aligned} \text{Sales(\$ millions)} &= 1.80 + .30(6) - 5.0(.12) \\ &= 1.8 + 1.8 - .6 \\ &= 3.00 \end{aligned}$$

or:

$$\text{Sales} = \$3{,}000{,}000$$

INSIGHT ▶ By using both variables, payroll and interest rates, Nodel now has a sales forecast of $3 million and a higher coefficient of correlation. This suggests a stronger relationship between the two variables and a more accurate estimate of sales.

LEARNING EXERCISE ▶ If interest rates were only 6%, what would be the sales forecast? [Answer: 1.8 + 1.8 − 5.0(.06) = 3.3, or $3,300,000.]

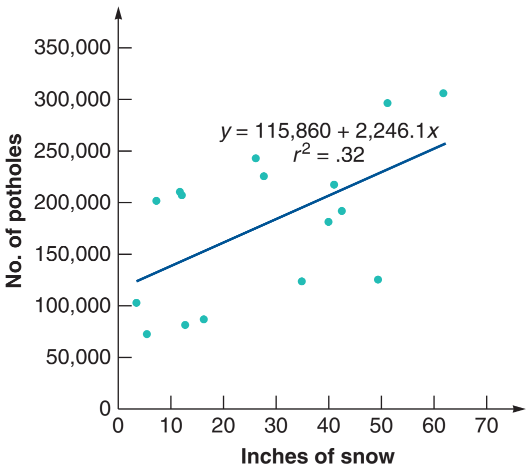
RELATED PROBLEMS ▶ 4.47, 4.49 (4.59 is available in MyOMLab)

The *OM in Action* box, "NYC's Potholes and Regression Analysis," provides an interesting example of one city's use of regression and multiple regression.

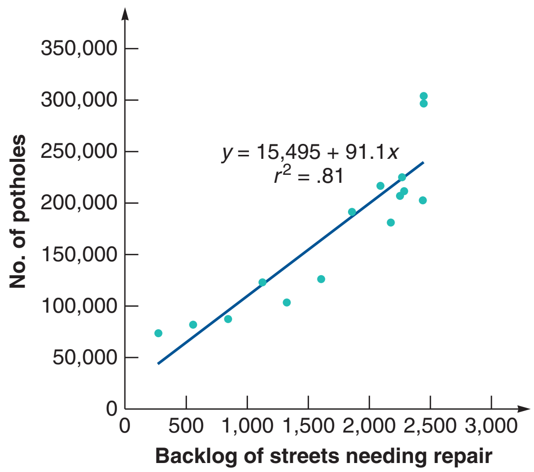
OM in Action — NYC's Potholes and Regression Analysis

New York is famous for many things, but one it does not like to be known for is its large and numerous potholes. David Letterman used to joke: "There is a pothole so big on 8th Avenue, it has its own Starbucks in it." When it comes to potholes, some years seem to be worse than others. The winter of 2014 was an exceptionally bad year. City workers filled a record 300,000 potholes during the first 4 months of the year. That's an astounding accomplishment.

But potholes are to some extent a measure of municipal competence—and they are costly. NYC's poor streets cost the average motorist an estimated $800 per year in repair work and new tires. There has been a steady and dramatic increase in potholes from around 70,000–80,000 in the 1990s to the devastatingly high 200,000–300,000 range in recent years. One theory is that bad weather causes the potholes. Using inches of snowfall as a measure of the severity of the winter, the graph below shows a plot of the number of potholes versus the inches of snow each winter.



Research showed that the city would need to resurface at least 1,000 miles of roads per year just to stay even with road deterioration. Any amount below that would contribute to a "gap" or backlog of streets needing repair. The graph below shows the plot of potholes versus the gap. With an r^2 of .81, there is a very strong relationship between the increase in the "gap" and the number of potholes. It is obvious that the real reason for the steady and substantial increase in the number of potholes is due to the increasing gap in road resurfacing.



A third model performs a regression analysis using the resurfacing gap and inches of snow as two independent variables and number of potholes as the dependent variable. That regression model's r^2 is .91.

$$\begin{aligned} \text{Potholes} = {} & 7{,}801.5 + 80.6 \times \text{Resurfacing gap} \\ & + 930.1 \times \text{Inches of snow} \end{aligned}$$

Sources: OR/MS Today (June, 2014) and *New York Daily News* (March 5, 2014).

Monitoring and Controlling Forecasts

Once a forecast has been completed, it should not be forgotten. No manager wants to be reminded that his or her forecast is horribly inaccurate, but a firm needs to determine why actual demand (or whatever variable is being examined) differed significantly from that projected. If the forecaster is accurate, that individual usually makes sure that everyone is aware of his or her talents. Very seldom does one read articles in *Fortune*, *Forbes*, or *The Wall Street Journal*, however, about money managers who are consistently off by 25% in their stock market forecasts.

Tracking signal

A measurement of how well a forecast is predicting actual values.

One way to monitor forecasts to ensure that they are performing well is to use a tracking signal. A **tracking signal** is a measurement of how well a forecast is predicting actual values. As forecasts are updated every week, month, or quarter, the newly available demand data are compared to the forecast values.

The tracking signal is computed as the cumulative error divided by the *mean absolute deviation (MAD)*:

$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}} \tag{4-18}$$

$$= \frac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\text{MAD}}$$

where $$\text{MAD} = \frac{\sum|\text{Actual} - \text{Forecast}|}{n}$$

STUDENT TIP

Using a tracking signal is a good way to make sure the forecasting system is continuing to do a good job.

as seen earlier, in Equation (4-5).

Positive tracking signals indicate that demand is *greater* than forecast. *Negative* signals mean that demand is *less* than forecast. A good tracking signal—that is, one with a low cumulative error—has about as much positive error as it has negative error. In other words, small deviations are okay, but positive and negative errors should balance one another so that the tracking signal centers closely around zero. A consistent tendency for forecasts to be greater or less than the actual values (that is, for a high absolute cumulative error) is called a **bias** error. Bias can occur if, for example, the wrong variables or trend line are used or if a seasonal index is misapplied.

Bias

A forecast that is consistently higher or consistently lower than actual values of a time series.

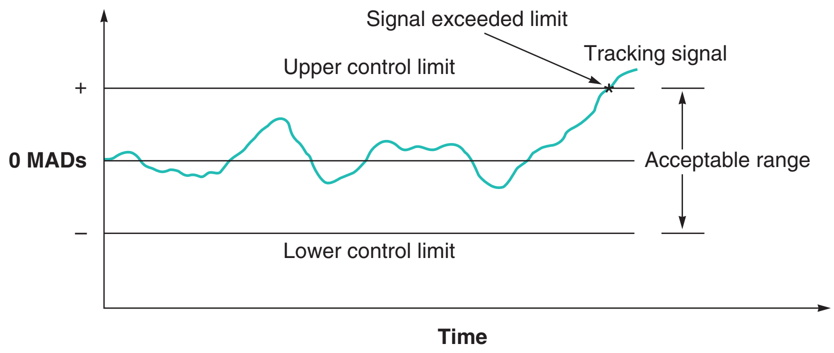
Once tracking signals are calculated, they are compared with predetermined control limits. When a tracking signal exceeds an upper or lower limit, there is a problem with the forecasting method, and management may want to reevaluate the way it forecasts demand. Figure 4.11 shows the graph of a tracking signal that is exceeding the range of acceptable variation. If the model being used is exponential smoothing, perhaps the smoothing constant needs to be readjusted.

LO 4.7 *Use* a tracking signal

How do firms decide what the upper and lower tracking limits should be? There is no single answer, but they try to find reasonable values—in other words, limits not so low as to be triggered with every small forecast error and not so high as to allow bad forecasts to be regularly overlooked. One MAD is equivalent to approximately .8 standard deviations,

Figure **4.11**

A Plot of Tracking Signals



± 2 MADs $= \pm 1.6$ standard deviations, ± 3 MADs $= \pm 2.4$ standard deviations, and ± 4 MADs $= \pm 3.2$ standard deviations. This fact suggests that for a forecast to be "in control," 89% of the errors are expected to fall within ± 2 MADs, 98% within ± 3 MADs, or 99.9% within ± 4 MADs.[3]

Example 16 shows how the tracking signal and cumulative error can be computed.

Example 16

COMPUTING THE TRACKING SIGNAL AT CARLSON'S BAKERY

Carlson's Bakery wants to evaluate performance of its croissant forecast.

APPROACH ▶ Develop a tracking signal for the forecast, and see if it stays within acceptable limits, which we define as ± 4 MADs.

SOLUTION ▶ Using the forecast and demand data for the past 6 quarters for croissant sales, we develop a tracking signal in the following table:

QUARTER	ACTUAL DEMAND	FORECAST DEMAND	ERROR	CUMULATIVE ERROR	ABSOLUTE FORECAST ERROR	CUMULATIVE ABSOLUTE FORECAST ERROR	MAD	TRACKING SIGNAL (CUMULATIVE ERROR/MAD)
1	90	100	−10	−10	10	10	10.0	−10/10 = −1
2	95	100	−5	−15	5	15	7.5	−15/7.5 = −2
3	115	100	+15	0	15	30	10.0	0/10 = 0
4	100	110	−10	−10	10	40	10.0	−10/10 = −1
5	125	110	+15	+5	15	55	11.0	+5/11 = +0.5
6	140	110	+30	+35	30	85	14.2	+35/14.2 = +2.5

$$\text{At the end of quarter 6, MAD} = \frac{\sum|\text{Forecast errors}|}{n} = \frac{85}{6} = 14.2$$

$$\text{and Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

INSIGHT ▶ Because the tracking signal drifted from −2 MAD to +2.5 MAD (between 1.6 and 2.0 standard deviations), we can conclude that it is within acceptable limits.

LEARNING EXERCISE ▶ If actual demand in quarter 6 was 130 (rather than 140), what would be the MAD and resulting tracking signal? [Answer: MAD for quarter 6 would be 12.5, and the tracking signal for period 6 would be 2 MADs.]

RELATED PROBLEMS ▶ 4.59, 4.60 (4.61c is available in MyOMLab)

Adaptive Smoothing

Adaptive forecasting refers to computer monitoring of tracking signals and self-adjustment if a signal passes a preset limit. For example, when applied to exponential smoothing, the α and β coefficients are first selected on the basis of values that minimize error forecasts and then adjusted accordingly whenever the computer notes an errant tracking signal. This process is called **adaptive smoothing**.

Adaptive smoothing
An approach to exponential smoothing forecasting in which the smoothing constant is automatically changed to keep errors to a minimum.

Focus Forecasting

Rather than adapt by choosing a smoothing constant, computers allow us to try a variety of forecasting models. Such an approach is called focus forecasting. **Focus forecasting** is based on two principles:

1. Sophisticated forecasting models are not always better than simple ones.
2. There is no single technique that should be used for all products or services.

Focus forecasting
Forecasting that tries a variety of computer models and selects the best one for a particular application.

Bernard Smith, inventory manager for American Hardware Supply, coined the term *focus forecasting*. Smith's job was to forecast quantities for 100,000 hardware products purchased by American's 21 buyers.[4] He found that buyers neither trusted nor understood the exponential smoothing model then in use. Instead, they used very simple approaches of their own. So Smith developed his new computerized system for selecting forecasting methods.

Smith chose to test seven forecasting methods. They ranged from the simple ones that buyers used (such as the naive approach) to statistical models. Every month, Smith applied the forecasts of all seven models to each item in stock. In these simulated trials, the forecast values were subtracted from the most recent actual demands, giving a simulated forecast error. The forecast method yielding the least error is selected by the computer, which then uses it to make next month's forecast. Although buyers still have an override capability, American Hardware finds that focus forecasting provides excellent results.

Forecasting in the Service Sector

STUDENT TIP
Forecasting at McDonald's, FedEx, and Walmart is as important and complex as it is for manufacturers such as Toyota and Dell.

Forecasting in the service sector presents some unusual challenges. A major technique in the retail sector is tracking demand by maintaining good short-term records. For instance, a barbershop catering to men expects peak flows on Fridays and Saturdays. Indeed, most barbershops are closed on Sunday and Monday, and many call in extra help on Friday and Saturday. A downtown restaurant, on the other hand, may need to track conventions and holidays for effective short-term forecasting.

Specialty Retail Shops Specialty retail facilities, such as flower shops, may have other unusual demand patterns, and those patterns will differ depending on the holiday. When Valentine's Day falls on a weekend, for example, flowers can't be delivered to offices, and those romantically inclined are likely to celebrate with outings rather than flowers. If a holiday falls on a Monday, some of the celebration may also take place on the weekend, reducing flower sales. However, when Valentine's Day falls in midweek, busy midweek schedules often make flowers the optimal way to celebrate. Because flowers for Mother's Day are to be delivered on Saturday or Sunday, this holiday forecast varies less. Due to special demand patterns, many service firms maintain records of sales, noting not only the day of the week but also unusual events, including the weather, so that patterns and correlations that influence demand can be developed.

VIDEO 4.2
Forecasting at Hard Rock Cafe

Fast-Food Restaurants Fast-food restaurants are well aware not only of weekly, daily, and hourly but even 15-minute variations in demands that influence sales. Therefore, detailed forecasts of demand are needed. Figure 4.12(a) shows the hourly forecast for a typical fast-food restaurant. Note the lunchtime and dinnertime peaks. This contrasts to the mid-morning and mid-afternoon peaks at FedEx's call center in Figure 4.12(b).

Firms like Taco Bell now use point-of-sale computers that track sales every quarter hour. Taco Bell found that a 6-week moving average was the forecasting technique that minimized its mean squared error (MSE) of these quarter-hour forecasts. Building this forecasting methodology into each of Taco Bell's 6,500 U.S. stores' computers, the model makes weekly projections of customer transactions. These in turn are used by store managers to schedule staff, who begin in 15-minute increments, not 1-hour blocks as in other industries. The forecasting model has been so successful that Taco Bell has increased customer service while documenting more than $50 million in labor cost savings in 4 years of use.

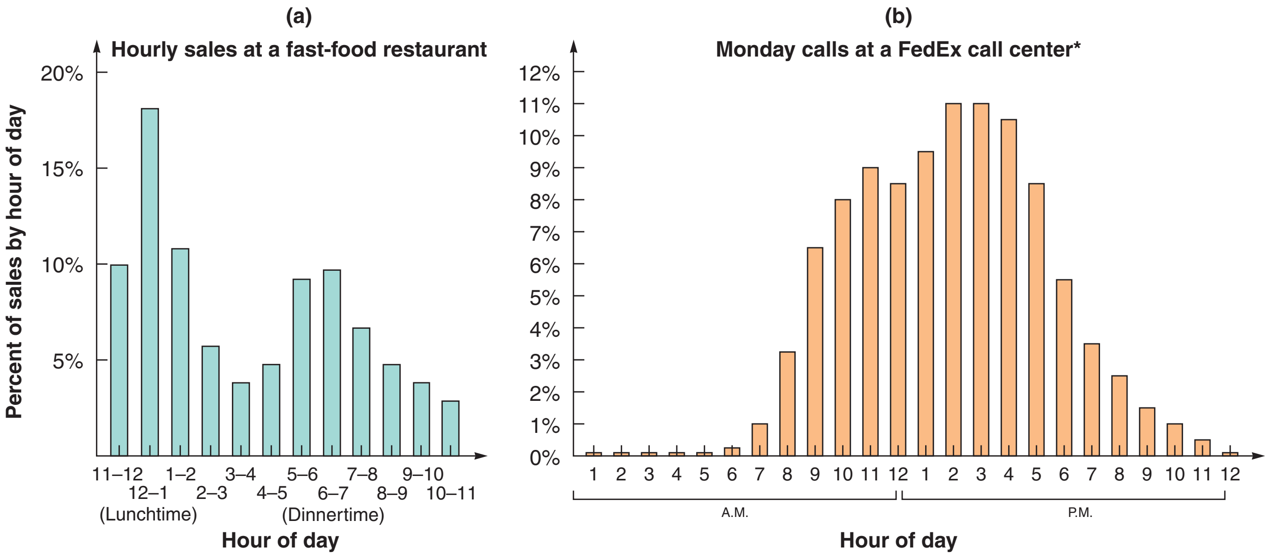


Figure **4.12**

Forecasts Are Unique: Note the Variations between (a) Hourly Sales at a Fast-Food Restaurant and (b) Hourly Call Volume at FedEx

*Based on historical data: see *Journal of Business Forecasting* (Winter 1999–2000): 6–11.

Summary

Forecasts are a critical part of the operations manager's function. Demand forecasts drive a firm's production, capacity, and scheduling systems and affect the financial, marketing, and personnel planning functions.

There are a variety of qualitative and quantitative forecasting techniques. Qualitative approaches employ judgment, experience, intuition, and a host of other factors that are difficult to quantify. Quantitative forecasting uses historical data and causal, or associative, relations to project future demands. The Rapid Review for this chapter summarizes the formulas we introduced in quantitative forecasting. Forecast calculations are seldom performed by hand. Most operations managers turn to software packages such as Forecast PRO, NCSS, Minitab, Systat, Statgraphics, SAS, or SPSS.

No forecasting method is perfect under all conditions. And even once management has found a satisfactory approach, it must still monitor and control forecasts to make sure errors do not get out of hand. Forecasting can often be a very challenging, but rewarding, part of managing.

Key Terms

Forecasting (p. 108)
Economic forecasts (p. 109)
Technological forecasts (p. 109)
Demand forecasts (p. 109)
Quantitative forecasts (p. 111)
Qualitative forecasts (p. 111)
Jury of executive opinion (p. 111)
Delphi method (p. 111)
Sales force composite (p. 111)
Market survey (p. 111)
Time series (p. 112)
Naive approach (p. 114)
Moving averages (p. 114)
Exponential smoothing (p. 116)
Smoothing constant (p. 116)
Mean absolute deviation (MAD) (p. 118)
Mean squared error (MSE) (p. 119)
Mean absolute percent error (MAPE) (p. 120)
Trend projection (p. 124)
Seasonal variations (p. 126)
Cycles (p. 131)
Linear-regression analysis (p. 131)
Standard error of the estimate (p. 133)
Coefficient of correlation (p. 134)
Coefficient of determination (p. 136)
Multiple regression (p. 136)
Tracking signal (p. 138)
Bias (p. 138)
Adaptive smoothing (p. 139)
Focus forecasting (p. 139)

Ethical Dilemma

We live in a society obsessed with test scores and maximum performance. Think of the SAT, ACT, GRE, GMAT, and LSAT. Though they take only a few hours, they are supposed to give schools and companies a snapshot of a student's abiding talents.

But these tests are often spectacularly bad at forecasting performance in the real world. The SAT does a decent job (r^2 = .12) of predicting the grades of a college freshman. It is, however, less effective at predicting achievement *after* graduation. LSAT scores bear virtually no correlation to career success as measured by income, life satisfaction, or public service.

Robert Kneschke/Fotolia

What does the r^2 mean in this context? Is it ethical for colleges to base admissions and financial aid decisions on scores alone? What role do these tests take at your own school?

Discussion Questions

1. What is a qualitative forecasting model, and when is its use appropriate?
2. Identify and briefly describe the two general forecasting approaches.
3. Identify the three forecasting time horizons. State an approximate duration for each.
4. Briefly describe the steps that are used to develop a forecasting system.
5. A skeptical manager asks what medium-range forecasts can be used for. Give the manager three possible uses/purposes.
6. Explain why such forecasting devices as moving averages, weighted moving averages, and exponential smoothing are not well suited for data series that have trends.
7. What is the basic difference between a weighted moving average and exponential smoothing?
8. What three methods are used to determine the accuracy of any given forecasting method? How would you determine whether time-series regression or exponential smoothing is better in a specific application?
9. Research and briefly describe the Delphi technique. How would it be used by an employer you have worked for?
10. What is the primary difference between a time-series model and an associative model?
11. Define *time series*.
12. What effect does the value of the smoothing constant have on the weight given to the recent values?
13. Explain the value of seasonal indices in forecasting. How are seasonal patterns different from cyclical patterns?
14. Which forecasting technique can place the most emphasis on recent values? How does it do this?
15. In your own words, explain adaptive forecasting.
16. What is the purpose of a tracking signal?
17. Explain, in your own words, the meaning of the correlation coefficient. Discuss the meaning of a negative value of the correlation coefficient.
18. What is the difference between a dependent and an independent variable?
19. Give examples of industries that are affected by seasonality. Why would these businesses want to filter out seasonality?
20. Give examples of industries in which demand forecasting is dependent on the demand for other products.
21. What happens to the ability to forecast for periods farther into the future?
22. CEO John Goodale, at Southern Illinois Power and Light, has been collecting data on demand for electric power in its western subregion for only the past 2 years. Those data are shown in the table below.

    To plan for expansion and to arrange to borrow power from neighboring utilities during peak periods, Goodale needs to be able to forecast demand for each month next year. However, the standard forecasting models discussed in this chapter will not fit the data observed for the 2 years.

    a) What are the weaknesses of the standard forecasting techniques as applied to this set of data?
    b) Because known models are not appropriate here, propose your own approach to forecasting. Although there is no perfect solution to tackling data such as these (in other words, there are no 100% right or wrong answers), justify your model.
    c) Forecast demand for each month next year using the model you propose.

DEMAND IN MEGAWATTS		
MONTH	LAST YEAR	THIS YEAR
January	5	17
February	6	14
March	10	20
April	13	23
May	18	30
June	15	38
July	23	44
August	26	41
September	21	33
October	15	23
November	12	26
December	14	17

Using Software in Forecasting

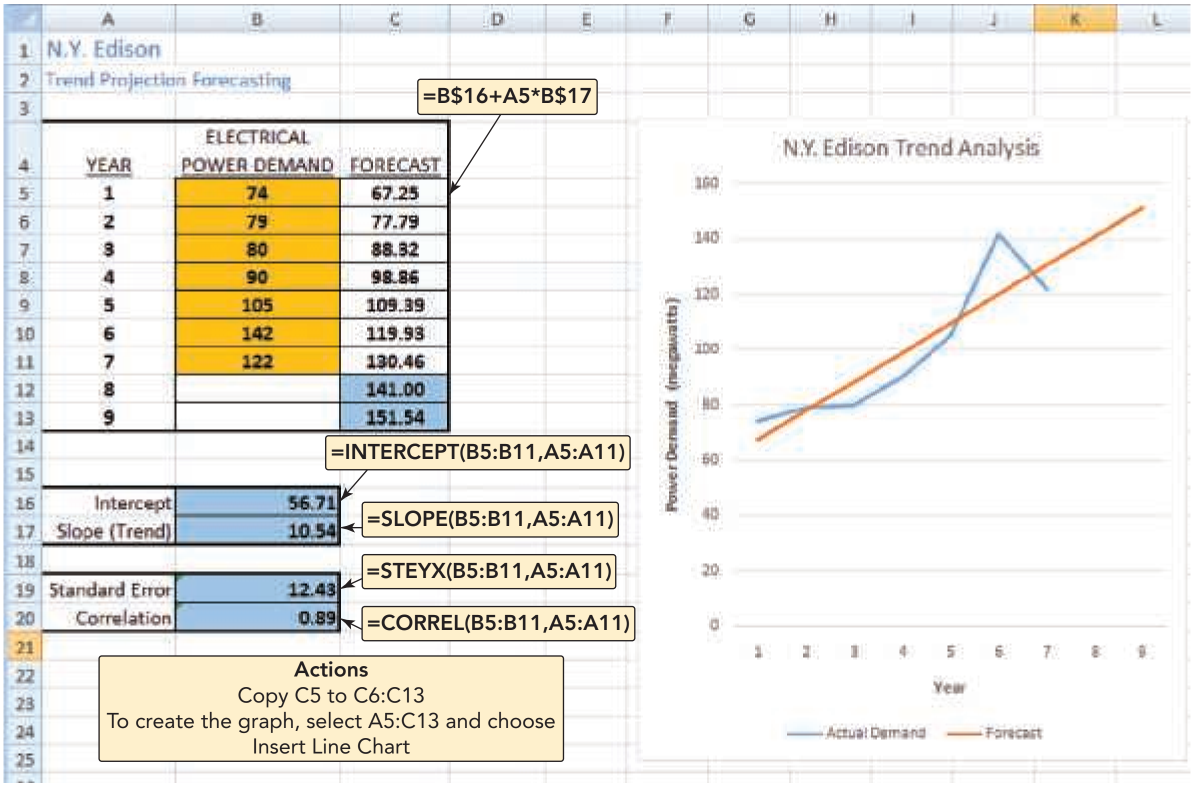
This section presents three ways to solve forecasting problems with computer software. First, you can create your own Excel spreadsheets to develop forecasts. Second, you can use the Excel OM software that comes with the text. Third, POM for Windows is another program that is located in MyOMLab.

CREATING YOUR OWN EXCEL SPREADSHEETS

Excel spreadsheets (and spreadsheets in general) are frequently used in forecasting. Exponential smoothing, trend analysis, and regression analysis (simple and multiple) are supported by built-in Excel functions.

Program 4.1 illustrates how to build an Excel forecast for the data in Example 8. The goal for N.Y. Edison is to create a trend analysis of the year 1 to year 7 data.

As an alternative, you may want to experiment with Excel's built-in regression analysis. To do so, under the **Data** menu bar selection choose **Data Analysis**, then **Regression**. Enter your *Y* and *X* data into two columns (say A and B). When the regression window appears, enter the *Y* and *X* ranges, then select **OK**. Excel offers several plots and tables to those interested in more rigorous analysis of regression problems.



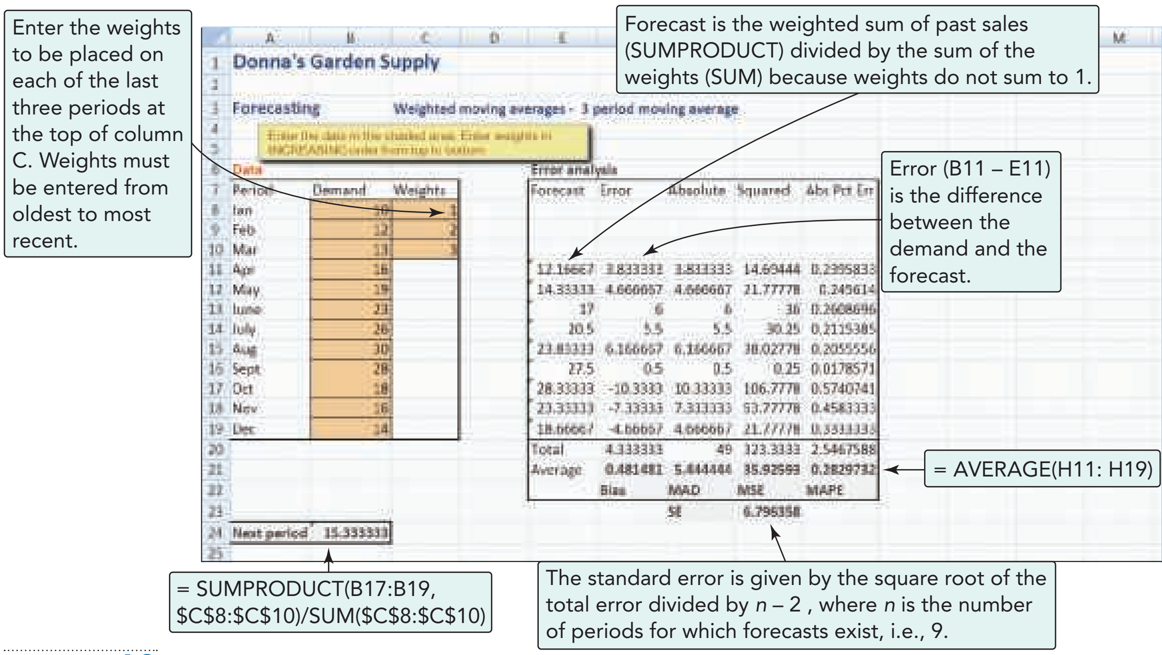
Program **4.1**

Using Excel to Develop Your Own Forecast, with Data from Example 8

USING EXCEL OM

Excel OM's forecasting module has five components: (1) moving averages, (2) weighted moving averages, (3) exponential smoothing, (4) regression (with one variable only), and (5) decomposition. Excel OM's error analysis is much more complete than that available with the Excel add-in.

Program 4.2 illustrates Excel OM's input and output, using Example 2's weighted-moving-average data.



Program **4.2**

Analysis of Excel OM's Weighted-Moving-Average Program, Using Data from Example 2 as Input

USING POM FOR WINDOWS

POM for Windows can project moving averages (both simple and weighted), handle exponential smoothing (both simple and trend adjusted), forecast with least squares trend projection, and solve linear regression (associative) models. A summary screen of error analysis and a graph of the data can also be generated. As a special example of exponential smoothing adaptive forecasting, when using an α of 0, POM for Windows will find the α value that yields the minimum MAD.

Appendix IV provides further details.

Solved Problems

Virtual Office Hours help is available in MyOMLab.

SOLVED PROBLEM 4.1

Sales of Volkswagen's popular Beetle have grown steadily at auto dealerships in Nevada during the past 5 years (see table below). The sales manager had predicted before the new model was introduced that first year sales would be 410 VWs. Using exponential smoothing with a weight of $\alpha = .30$, develop forecasts for years 2 through 6.

YEAR	SALES	FORECAST
1	450	410
2	495	
3	518	
4	563	
5	584	
6	?	

SOLUTION

YEAR	FORECAST
1	410.0
2	422.0 = 410 + .3 (450 − 410)
3	443.9 = 422 + .3 (495 − 422)
4	466.1 = 443.9 + .3 (518 − 443.9)
5	495.2 = 466.1 + .3 (563 − 466.1)
6	521.8 = 495.2 + .3 (584 − 495.2)

SOLVED PROBLEM 4.2

In Example 7, we applied trend-adjusted exponential smoothing to forecast demand for a piece of pollution-control equipment for months 2 and 3 (out of 9 months of data provided). Let us now continue this process for month 4. We want to confirm the forecast for month 4 shown in Table 4.2 (p. 123) and Figure 4.3 (p. 123).

For month 4, $A_4 = 19$, with $\alpha = .2$, and $\beta = .4$.

SOLUTION

$$
\begin{aligned}
F_4 &= \alpha A_3 + (1 - \alpha)(F_3 + T_3) \\
&= (.2)(20) + (1 - .2)(15.18 + 2.10) \\
&= 4.0 + (.8)(17.28) \\
&= 4.0 + 13.82 \\
&= 17.82 \\
T_4 &= \beta(F_4 - F_3) + (1 - \beta)T_3 \\
&= (.4)(17.82 - 15.18) + (1 - .4)(2.10) \\
&= (.4)(2.64) + (.6)(2.10) \\
&= 1.056 + 1.26 \\
&= 2.32 \\
FIT_4 &= 17.82 + 2.32 \\
&= 20.14
\end{aligned}
$$

SOLVED PROBLEM 4.3

Sales of hair dryers at the Walgreens stores in Youngstown, Ohio, over the past 4 months have been 100, 110, 120, and 130 units (with 130 being the most recent sales).

Develop a moving-average forecast for next month, using these three techniques:

a) 3-month moving average.
b) 4-month moving average.
c) Weighted 4-month moving average with the most recent month weighted 4, the preceding month 3, then 2, and the oldest month weighted 1.
d) If next month's sales turn out to be 140 units, forecast the following month's sales (months) using a 4-month moving average.

SOLUTION

a) 3-month moving average

$$= \frac{110 + 120 + 130}{3} = \frac{360}{3} = 120 \text{ dryers}$$

b) 4-month moving average

$$= \frac{100 + 110 + 120 + 130}{4} = \frac{460}{4} = 115 \text{ dryers}$$

c) Weighted moving average

$$= \frac{4(130) + 3(120) + 2(110) + 1(100)}{10}$$

$$= \frac{1{,}200}{10} = 120 \text{ dryers}$$

d) *Now* the four most recent sales are 110, 120, 130, and 140.

$$\text{4-month moving average} = \frac{110 + 120 + 130 + 140}{4}$$

$$= \frac{500}{4} = 125 \text{ dryers}$$

We note, of course, the lag in the forecasts, as the moving-average method does not immediately recognize trends.

SOLVED PROBLEM 4.4

The following data come from regression line projections:

PERIOD	FORECAST VALUES	ACTUAL VALUES
1	410	406
2	419	423
3	428	423
4	435	440

Compute the MAD and MSE.

SOLUTION

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n}$$
$$= \frac{|406-410| + |423-419| + |423-428| + |440-435|}{4}$$
$$= \frac{4+4+5+5}{4} = \frac{18}{4} = 4.5$$

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n}$$
$$= \frac{(406-410)^2+(423-419)^2+(423-428)^2+(440-435)^2}{4}$$
$$= \frac{4^2+4^2+5^2+5^2}{4} = \frac{16+16+25+25}{4} = 20.5$$

SOLVED PROBLEM 4.5

Room registrations in the Toronto Towers Plaza Hotel have been recorded for the past 9 years. To project future occupancy, management would like to determine the mathematical trend of guest registration. This estimate will help the hotel determine whether future expansion will be needed. Given the following time-series data, develop a regression equation relating registrations to time (e.g., a trend equation). Then forecast year 11 registrations. Room registrations are in the thousands:

Year 1: 17	Year 2: 16	Year 3: 16	Year 4: 21	Year 5: 20
Year 6: 20	Year 7: 23	Year 8: 25	Year 9: 24	

SOLUTION

YEAR	REGISTRANTS, y (IN THOUSANDS)	x^2	xy
1	17	1	17
2	16	4	32
3	16	9	48
4	21	16	84
5	20	25	100
6	20	36	120
7	23	49	161
8	25	64	200
9	24	81	216
$\sum x = 45$	$\sum y = 182$	$\sum x^2 = 285$	$\sum xy = 978$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{978 - (9)(5)(20.22)}{285 - (9)(25)}$$
$$= \frac{978 - 909.9}{285 - 225} = \frac{68.1}{60} = 1.135$$

$a = \bar{y} - b\bar{x} = 20.22 - (1.135)(5) = 20.22 - 5.675 = 14.545$

$\hat{y} = \text{(registrations)} = 14.545 + 1.135x$

The projection of registrations in year 11 is:

$\hat{y} = 14.545 + (1.135)(11) = 27.03$ or 27,030 guests in year 11.

SOLVED PROBLEM 4.6

Quarterly demand for Ford F150 pickups at a New York auto dealer is forecast with the equation:

$$\hat{y} = 10 + 3x$$

where x = quarters, and:

Quarter I of year 1 = 0
Quarter II of year 1 = 1
Quarter III of year 1 = 2
Quarter IV of year 1 = 3
Quarter I of year 2 = 4
and so on

and:

$$\hat{y} = \text{quarterly demand}$$

The demand for trucks is seasonal, and the indices for Quarters I, II, III, and IV are 0.80, 1.00, 1.30, and 0.90, respectively. Forecast demand for each quarter of year 3. Then, seasonalize each forecast to adjust for quarterly variations.

SOLUTION

Quarter II of year 2 is coded $x = 5$; Quarter III of year 2, $x = 6$; and Quarter IV of year 2, $x = 7$. Hence, Quarter I of year 3 is coded $x = 8$; Quarter II, $x = 9$; and so on.

$\hat{y}$ (Year 3 Quarter I) = 10 + 3(8) = 34
$\hat{y}$ (Year 3 Quarter II) = 10 + 3(9) = 37
$\hat{y}$ (Year 3 Quarter III) = 10 + 3(10) = 40
$\hat{y}$ (Year 3 Quarter IV) = 10 + 3(11) = 43

Adjusted forecast = (.80)(34) = 27.2
Adjusted forecast = (1.00)(37) = 37
Adjusted forecast = (1.30)(40) = 52
Adjusted forecast = (.90)(43) = 38.7

SOLVED PROBLEM 4.7

Cengiz Haksever runs an Istanbul high-end jewelry shop. He advertises weekly in local Turkish newspapers and is thinking of increasing his ad budget. Before doing so, he decides to evaluate the past effectiveness of these ads. Five weeks are sampled, and the data are shown in the table below:

SALES ($1,000s)	AD BUDGET THAT WEEK ($100s)
11	5
6	3
10	7
6	2
12	8

Develop a regression model to help Cengiz evaluate his advertising.

SOLUTION

We apply the least-squares regression model as we did in Example 12.

SALES, y	ADVERTISING, x	x^2	xy
11	5	25	55
6	3	9	18
10	7	49	70
6	2	4	12
12	8	64	96
$\Sigma y = 45$	$\Sigma x = 25$	$\Sigma x^2 = 151$	$\Sigma xy = 251$
$\bar{y} = \frac{45}{5} = 9$	$\bar{x} = \frac{25}{5} = 5$		

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{251 - (5)(5)(9)}{151 - (5)(5^2)}$$

$$= \frac{251 - 225}{151 - 125} = \frac{26}{26} = 1$$

$$a = \bar{y} - b\bar{x} = 9 - (1)(5) = 4$$

So the regression model is $\hat{y} = 4 + 1x$, or

Sales (in \$1,000s) = 4 + 1 (Ad budget in \$100s)

This means that for each 1-unit increase in x (or \$100 in ads), sales increase by 1 unit (or \$1,000).

SOLVED PROBLEM 4.8

Using the data in Solved Problem 4.7, find the coefficient of determination, r^2, for the model.

SOLUTION

To find r^2, we need to also compute Σy^2.

$$\Sigma y^2 = 11^2 + 6^2 + 10^2 + 6^2 + 12^2$$
$$= 121 + 36 + 100 + 36 + 144 = 437$$

The next step is to find the coefficient of correlation, r:

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{5(251) - (25)(45)}{\sqrt{[5(151) - (25)^2][5(437) - (45)^2]}}$$

$$= \frac{1{,}255 - 1{,}125}{\sqrt{(130)(160)}} = \frac{130}{\sqrt{20{,}800}} = \frac{130}{144.22}$$

$$= .9014$$

Thus, $r^2 = (.9014)^2 = .8125$, meaning that about 81% of the variability in sales can be explained by the regression model with advertising as the independent variable.

Problems

Note: Px means the problem may be solved with POM for Windows and/or Excel OM.

Problems 4.1–4.42 relate to Time-Series Forecasting

• **4.1** The following gives the number of pints of type B blood used at Woodlawn Hospital in the past 6 weeks:

WEEK OF	PINTS USED
August 31	360
September 7	389
September 14	410
September 21	381
September 28	368
October 5	374

a) Forecast the demand for the week of October 12 using a 3-week moving average.

b) Use a 3-week weighted moving average, with weights of .1, .3, and .6, using .6 for the most recent week. Forecast demand for the week of October 12.

c) Compute the forecast for the week of October 12 using exponential smoothing with a forecast for August 31 of 360 and $\alpha = .2$. Px

•• **4.2**

YEAR	1	2	3	4	5	6	7	8	9	10	11
DEMAND	7	9	5	9	13	8	12	13	9	11	7

a) Plot the above data on a graph. Do you observe any trend, cycles, or random variations?

b) Starting in year 4 and going to year 12, forecast demand using a 3-year moving average. Plot your forecast on the same graph as the original data.

c) Starting in year 4 and going to year 12, forecast demand using a 3-year moving average with weights of .1, .3, and .6, using .6 for the most recent year. Plot this forecast on the same graph.
d) As you compare forecasts with the original data, which seems to give the better results? PX

•• **4.3** Refer to Problem 4.2. Develop a forecast for years 2 through 12 using exponential smoothing with $\alpha = .4$ and a forecast for year 1 of 6. Plot your new forecast on a graph with the actual data and the naive forecast. Based on a visual inspection, which forecast is better? PX

• **4.4** A check-processing center uses exponential smoothing to forecast the number of incoming checks each month. The number of checks received in June was 40 million, while the forecast was 42 million. A smoothing constant of .2 is used.
a) What is the forecast for July?
b) If the center received 45 million checks in July, what would be the forecast for August?
c) Why might this be an inappropriate forecasting method for this situation? PX

•• **4.5** The Carbondale Hospital is considering the purchase of a new ambulance. The decision will rest partly on the anticipated mileage to be driven next year. The miles driven during the past 5 years are as follows:

YEAR	MILEAGE
1	3,000
2	4,000
3	3,400
4	3,800
5	3,700

a) Forecast the mileage for next year (6th year) using a 2-year moving average.
b) Find the MAD based on the 2-year moving average. (*Hint:* You will have only 3 years of matched data.)
c) Use a weighted 2-year moving average with weights of .4 and .6 to forecast next year's mileage. (The weight of .6 is for the most recent year.) What MAD results from using this approach to forecasting? (*Hint:* You will have only 3 years of matched data.)
d) Compute the forecast for year 6 using exponential smoothing, an initial forecast for year 1 of 3,000 miles, and $\alpha = .5$. PX

•• **4.6** The monthly sales for Yazici Batteries, Inc., were as follows:

MONTH	SALES
January	20
February	21
March	15
April	14
May	13
June	16
July	17
August	18
September	20
October	20
November	21
December	23

a) Plot the monthly sales data.
b) Forecast January sales using each of the following:
   i) Naive method.
   ii) A 3-month moving average.
   iii) A 6-month weighted average using .1, .1, .1, .2, .2, and .3, with the heaviest weights applied to the most recent months.
   iv) Exponential smoothing using an $\alpha = .3$ and a September forecast of 18.
   v) A trend projection.
c) With the data given, which method would allow you to forecast next March's sales? PX

•• **4.7** The actual demand for the patients at Omaha Emergency Medical Clinic for the first 6 weeks of this year follows:

WEEK	ACTUAL NO. OF PATIENTS
1	65
2	62
3	70
4	48
5	63
6	52

Clinic administrator Marc Schniederjans wants you to forecast patient demand at the clinic for week 7 by using this data. You decide to use a weighted moving average method to find this forecast. Your method uses four actual demand levels, with weights of 0.333 on the present period, 0.25 one period ago, 0.25 two periods ago, and 0.167 three periods ago.
a) What is the value of your forecast? PX
b) If instead the weights were 20, 15, 15, and 10, respectively, how would the forecast change? Explain why.
c) What if the weights were 0.40, 0.30, 0.20, and 0.10, respectively? Now what is the forecast for week 7?

• **4.8** Daily high temperatures in St. Louis for the last week were as follows: 93, 94, 93, 95, 96, 88, 90 (yesterday).
a) Forecast the high temperature today, using a 3-day moving average.
b) Forecast the high temperature today, using a 2-day moving average.
c) Calculate the mean absolute deviation based on a 2-day moving average.
d) Compute the mean squared error for the 2-day moving average.
e) Calculate the mean absolute percent error for the 2-day moving average. PX

••• **4.9** Lenovo uses the ZX-81 chip in some of its laptop computers. The prices for the chip during the past 12 months were as follows:

MONTH	PRICE PER CHIP	MONTH	PRICE PER CHIP
January	\$1.80	July	1.80
February	1.67	August	1.83
March	1.70	September	1.70
April	1.85	October	1.65
May	1.90	November	1.70
June	1.87	December	1.75

a) Use a 2-month moving average on all the data and plot the averages and the prices.
b) Use a 3-month moving average and add the 3-month plot to the graph created in part (a).
c) Which is better (using the mean absolute deviation): the 2-month average or the 3-month average?
d) Compute the forecasts for each month using exponential smoothing, with an initial forecast for January of $1.80. Use $\alpha = .1$, then $\alpha = .3$, and finally $\alpha = .5$. Using MAD, which α is the best?

• • **4.10** Data collected on the yearly registrations for a Six Sigma seminar at the Quality College are shown in the following table:

YEAR	1	2	3	4	5	6	7	8	9	10	11
REGISTRATIONS (000)	4	6	4	5	10	8	7	9	12	14	15

a) Develop a 3-year moving average to forecast registrations from year 4 to year 12.
b) Estimate demand again for years 4 to 12 with a 3-year weighted moving average in which registrations in the most recent year are given a weight of 2, and registrations in the other 2 years are each given a weight of 1.
c) Graph the original data and the two forecasts. Which of the two forecasting methods seems better?

• **4.11** Use exponential smoothing with a smoothing constant of 0.3 to forecast the registrations at the seminar given in Problem 4.10. To begin the procedure, assume that the forecast for year 1 was 5,000 people signing up.
a) What is the MAD?
b) What is the MSE?

• • **4.12** Consider the following actual and forecast demand levels for Big Mac hamburgers at a local McDonald's restaurant:

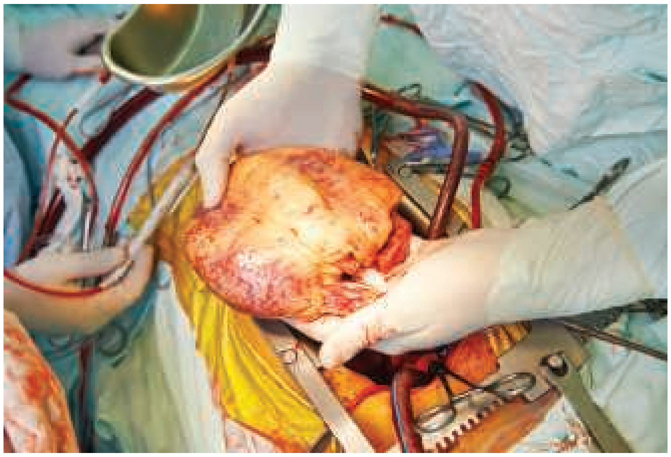
DAY	ACTUAL DEMAND	FORECAST DEMAND
Monday	88	88
Tuesday	72	88
Wednesday	68	84
Thursday	48	80
Friday		

The forecast for Monday was derived by observing Monday's demand level and setting Monday's forecast level equal to this demand level. Subsequent forecasts were derived by using exponential smoothing with a smoothing constant of 0.25. Using this exponential smoothing method, what is the forecast for Big Mac demand for Friday?

• • • **4.13** As you can see in the following table, demand for heart transplant surgery at Washington General Hospital has increased steadily in the past few years:

YEAR	1	2	3	4	5	6
HEART TRANSPLANTS	45	50	52	56	58	?

The director of medical services predicted 6 years ago that demand in year 1 would be 41 surgeries.


Dmitry Kalinovsky/Shutterstock

a) Use exponential smoothing, first with a smoothing constant of .6 and then with one of .9, to develop forecasts for years 2 through 6.
b) Use a 3-year moving average to forecast demand in years 4, 5, and 6.
c) Use the trend-projection method to forecast demand in years 1 through 6.
d) With MAD as the criterion, which of the four forecasting methods is best?

• • **4.14** Following are two weekly forecasts made by two different methods for the number of gallons of gasoline, in thousands, demanded at a local gasoline station. Also shown are actual demand levels, in thousands of gallons.

	FORECASTS		
WEEK	METHOD 1	METHOD 2	ACTUAL DEMAND
1	0.90	0.80	0.70
2	1.05	1.20	1.00
3	0.95	0.90	1.00
4	1.20	1.11	1.00

What are the MAD and MSE for each method?

• **4.15** Refer to Solved Problem 4.1 on page 144.
a) Use a 3-year moving average to forecast the sales of Volkswagen Beetles in Nevada through year 6.
b) What is the MAD?
c) What is the MSE?

• **4.16** Refer to Solved Problem 4.1 on page 144.
a) Using the trend projection (regression) method, develop a forecast for the sales of Volkswagen Beetles in Nevada through year 6.
b) What is the MAD?
c) What is the MSE?

• **4.17** Refer to Solved Problem 4.1 on page 144. Using smoothing constants of .6 and .9, develop forecasts for the sales of VW Beetles. What effect did the smoothing constant have on the forecast? Use MAD to determine which of the three smoothing constants (.3, .6, or .9) gives the most accurate forecast.

• • • • **4.18** Consider the following actual (A_t) and forecast (F_t) demand levels for a commercial multiline telephone at Office Max:

TIME PERIOD, t	ACTUAL DEMAND, A_t	FORECAST DEMAND, F_t
1	50	50
2	42	50
3	56	48
4	46	50
5		

The first forecast, F_1, was derived by observing A_1 and setting F_1 equal to A_1. Subsequent forecast averages were derived by exponential smoothing. Using the exponential smoothing method, find the forecast for time period 5. (*Hint:* You need to first find the smoothing constant, α.)

• • • **4.19** Income at the architectural firm Spraggins and Yunes for the period February to July was as follows:

MONTH	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY
Income (in $ thousand)	70.0	68.5	64.8	71.7	71.3	72.8

Use trend-adjusted exponential smoothing to forecast the firm's August income. Assume that the initial forecast average for February is $65,000 and the initial trend adjustment is 0. The smoothing constants selected are $\alpha = .1$ and $\beta = .2$. PX

• • • **4.20** Resolve Problem 4.19 with $\alpha = .1$ and $\beta = .8$. Using MSE, determine which smoothing constants provide a better forecast. PX

• **4.21** Refer to the trend-adjusted exponential smoothing illustration in Example 7 on pages 122–123. Using $\alpha = .2$ and $\beta = .4$, we forecast sales for 9 months, showing the detailed calculations for months 2 and 3. In Solved Problem 4.2, we continued the process for month 4.

In this problem, show your calculations for months 5 and 6 for F_t, T_t, and FIT_t. PX

• **4.22** Refer to Problem 4.21. Complete the trend-adjusted exponential-smoothing forecast computations for periods 7, 8, and 9. Confirm that your numbers for F_t, T_t, and FIT_t match those in Table 4.2 (p. 123). PX

• • **4.23** Sales of quilt covers at Bud Banis's department store in Carbondale over the past year are shown below. Management prepared a forecast using a combination of exponential smoothing and its collective judgment for the 4 months (March, April, May, and June):

MONTH	UNIT SALES	MANAGEMENT'S FORECAST
July	100	
August	93	
September	96	
October	110	
November	124	
December	119	
January	92	
February	83	
March	101	120
April	96	114
May	89	110
June	108	108

a) Compute MAD and MAPE for management's technique.
b) Do management's results outperform (i.e., have smaller MAD and MAPE than) a naive forecast?
c) Which forecast do you recommend, based on lower forecast error? PX

• **4.24** The following gives the number of accidents that occurred on Florida State Highway 101 during the past 4 months:

MONTH	NUMBER OF ACCIDENTS
January	30
February	40
March	60
April	90

Forecast the number of accidents that will occur in May, using least-squares regression to derive a trend equation. PX

• **4.25** In the past, Peter Kelle's tire dealership in Baton Rouge sold an average of 1,000 radials each year. In the past 2 years, 200 and 250, respectively, were sold in fall, 350 and 300 in winter, 150 and 165 in spring, and 300 and 285 in summer. With a major expansion planned, Kelle projects sales next year to increase to 1,200 radials. What will be the demand during each season?

• • **4.26** George Kyparisis owns a company that manufactures sailboats. Actual demand for George's sailboats during each of the past four seasons was as follows:

	YEAR			
SEASON	1	2	3	4
Winter	1,400	1,200	1,000	900
Spring	1,500	1,400	1,600	1,500
Summer	1,000	2,100	2,000	1,900
Fall	600	750	650	500

George has forecasted that annual demand for his sailboats in year 5 will equal 5,600 sailboats. Based on this data and the multiplicative seasonal model, what will the demand level be for George's sailboats in the spring of year 5?

• • **4.27** Attendance at Orlando's newest Disneylike attraction, Lego World, has been as follows:

QUARTER	GUESTS (IN THOUSANDS)	QUARTER	GUESTS (IN THOUSANDS)
Winter Year 1	73	Summer Year 2	124
Spring Year 1	104	Fall Year 2	52
Summer Year 1	168	Winter Year 3	89
Fall Year 1	74	Spring Year 3	146
Winter Year 2	65	Summer Year 3	205
Spring Year 2	82	Fall Year 3	98

Compute seasonal indices using all of the data. PX

• **4.28** North Dakota Electric Company estimates its demand trend line (in millions of kilowatt hours) to be:

$$D = 77 + 0.43Q$$

where Q refers to the sequential quarter number and $Q = 1$ for winter of Year 1. In addition, the multiplicative seasonal factors are as follows:

QUARTER	FACTOR (INDEX)
Winter	.8
Spring	1.1
Summer	1.4
Fall	.7

Forecast energy use for the four quarters of year 26 (namely quarters 101 to 104), beginning with winter.

• **4.29** The number of disk drives (in millions) made at a plant in Taiwan during the past 5 years follows:

YEAR	DISK DRIVES
1	140
2	160
3	190
4	200
5	210

a) Forecast the number of disk drives to be made next year, using linear regression.
b) Compute the mean squared error (MSE) when using linear regression.
c) Compute the mean absolute percent error (MAPE). PX

•• **4.30** Dr. Lillian Fok, a New Orleans psychologist, specializes in treating patients who are agoraphobic (i.e., afraid to leave their homes). The following table indicates how many patients Dr. Fok has seen each year for the past 10 years. It also indicates what the robbery rate was in New Orleans during the same year:

YEAR	1	2	3	4	5	6	7	8	9	10
NUMBER OF PATIENTS	36	33	40	41	40	55	60	54	58	61
ROBBERY RATE PER 1,000 POPULATION	58.3	61.1	73.4	75.7	81.1	89.0	101.1	94.8	103.3	116.2

Using trend (linear regression) analysis, predict the number of patients Dr. Fok will see in years 11 and 12 as a function of time. How well does the model fit the data? PX

••• **4.31** Emergency calls to the 911 system of Durham, North Carolina, for the past 24 weeks are shown in the following table:

WEEK	1	2	3	4	5	6	7	8	9	10	11	12
CALLS	50	35	25	40	45	35	20	30	35	20	15	40
WEEK	13	14	15	16	17	18	19	20	21	22	23	24
CALLS	55	35	25	55	55	40	35	60	75	50	40	65

a) Compute the exponentially smoothed forecast of calls for each week. Assume an initial forecast of 50 calls in the first week, and use $\alpha = .2$. What is the forecast for week 25?
b) Reforecast each period using $\alpha = .6$.
c) Actual calls during week 25 were 85. Which smoothing constant provides a superior forecast? Explain and justify the measure of error you used. PX

••• **4.32** Using the 911 call data in Problem 4.31, forecast calls for weeks 2 through 25 with a trend-adjusted exponential smoothing model. Assume an initial forecast for 50 calls for week 1 and an initial trend of zero. Use smoothing constants of $\alpha = .3$ and $\beta = .2$. Is this model better than that of Problem 4.31? What adjustment might be useful for further improvement? (Again, assume that actual calls in week 25 were 85.) PX

••• **4.33** Storrs Cycles has just started selling the new Cyclone mountain bike, with monthly sales as shown in the table. First, co-owner Bob Day wants to forecast by exponential smoothing by initially setting February's forecast equal to January's sales with $\alpha = .1$. Co-owner Sherry Snyder wants to use a three-period moving average.

	SALES	BOB	SHERRY	BOB'S ERROR	SHERRY'S ERROR
JANUARY	400	—			
FEBRUARY	380	400			
MARCH	410				
APRIL	375				
MAY					

a) Is there a strong linear trend in sales over time?
b) Fill in the table with what Bob and Sherry each forecast for May and the earlier months, as relevant.
c) Assume that May's actual sales figure turns out to be 405. Complete the table's columns and then calculate the mean absolute deviation for both Bob's and Sherry's methods.
d) Based on these calculations, which method seems more accurate? PX

•••• **4.34** Boulanger Savings and Loan is proud of its long tradition in Winter Park, Florida. Begun by Michelle Boulanger 22 years after World War II, the S&L has bucked the trend of financial and liquidity problems that has repeatedly plagued the industry. Deposits have increased slowly but surely over the years, despite recessions in 1983, 1988, 1991, 2001, and 2010. Ms. Boulanger believes it is necessary to have a long-range strategic plan for her firm, including a 1-year forecast and preferably even a 5-year forecast of deposits. She examines the past deposit data and also peruses Florida's gross state product (GSP) over the same 44 years. (GSP is analogous to gross national product [GNP] but on the state level.) The resulting data are in the following table.

YEAR	DEPOSITS[a]	GSP[b]	YEAR	DEPOSITS[a]	GSP[b]
1	.25	.4	13	.50	1.2
2	.24	.4	14	.95	1.2
3	.24	.5	15	1.70	1.2
4	.26	.7	16	2.3	1.6
5	.25	.9	17	2.8	1.5
6	.30	1.0	18	2.8	1.6
7	.31	1.4	19	2.7	1.7
8	.32	1.7	20	3.9	1.9
9	.24	1.3	21	4.9	1.9
10	.26	1.2	22	5.3	2.3
11	.25	1.1	23	6.2	2.5
12	.33	.9	24	4.1	2.8

(continued)

YEAR	DEPOSITS[a]	GSP[b]	YEAR	DEPOSITS[a]	GSP[b]
25	4.5	2.9	35	31.1	4.1
26	6.1	3.4	36	31.7	4.1
27	7.7	3.8	37	38.5	4.0
28	10.1	4.1	38	47.9	4.5
29	15.2	4.0	39	49.1	4.6
30	18.1	4.0	40	55.8	4.5
31	24.1	3.9	41	70.1	4.6
32	25.6	3.8	42	70.9	4.6
33	30.3	3.8	43	79.1	4.7
34	36.0	3.7	44	94.0	5.0

[a] In $ millions.
[b] In $ billions.

a) Using exponential smoothing, with $\alpha = .6$, then trend analysis, and finally linear regression, discuss which forecasting model fits best for Boulanger's strategic plan. Justify the selection of one model over another.
b) Carefully examine the data. Can you make a case for excluding a portion of the information? Why? Would that change your choice of model? PX

Additional problems **4.35–4.42** *are available in* MyOMLab.

Problems 4.43–4.58 relate to Associative Forecasting Methods

•• **4.43** Mark Gershon, owner of a musical instrument distributorship, thinks that demand for guitars may be related to the number of television appearances by the popular group Maroon 5 during the previous month. Mark has collected the data shown in the following table:

DEMAND FOR GUITARS	3	6	7	5	10	7
MAROON 5 TV APPEARANCES	3	4	7	6	8	5

a) Graph these data to see whether a linear equation might describe the relationship between the group's television shows and guitar sales.
b) Use the least-squares regression method to derive a forecasting equation.
c) What is your estimate for guitar sales if Maroon 5 performed on TV nine times last month?
d) What are the correlation coefficient (r) and the coefficient of determination (r^2) for this model, and what do they mean? PX

• **4.44** Lori Cook has developed the following forecasting model:

$$\hat{y} = 36 + 4.3x$$

where $\hat{y}$ = demand for Kool Air conditioners and
x = the outside temperature (°F) PX

a) Forecast demand for the Kool Air when the temperature is 70°F.
b) What is demand when the temperature is 80°F?
c) What is demand when the temperature is 90°F? PX

•• **4.45** Café Michigan's manager, Gary Stark, suspects that demand for mocha latte coffees depends on the price being charged. Based on historical observations, Gary has gathered the following data, which show the numbers of these coffees sold over six different price values:

PRICE	NUMBER SOLD
$2.70	760
$3.50	510
$2.00	980
$4.20	250
$3.10	320
$4.05	480

Using these data, how many mocha latte coffees would be forecast to be sold according to simple linear regression if the price per cup were $2.80? PX

• **4.46** The following data relate the sales figures of the bar in Mark Kaltenbach's small bed-and-breakfast inn in Portand, to the number of guests registered that week:

WEEK	GUESTS	BAR SALES
1	16	$330
2	12	270
3	18	380
4	14	300

a) Perform a linear regression that relates bar sales to guests (not to time).
b) If the forecast is for 20 guests next week, what are the sales expected to be? PX

• **4.47** The number of auto accidents in Athens, Ohio, is related to the regional number of registered automobiles in thousands (X_1), alcoholic beverage sales in $10,000s ($X_2$), and rainfall in inches (X_3). Furthermore, the regression formula has been calculated as:

$$Y = a + b_1X_1 + b_2X_2 + b_3X_3$$

where

Y = number of automobile accidents
$a = 7.5$
$b_1 = 3.5$
$b_2 = 4.5$
$b_3 = 2.5$

Calculate the expected number of automobile accidents under conditions a, b, and c:

	X_1	X_2	X_3
(a)	2	3	0
(b)	3	5	1
(c)	4	7	2

•• **4.48** Rhonda Clark, a Slippery Rock, Pennsylvania, real estate developer, has devised a regression model to help determine residential housing prices in northwestern Pennsylvania. The model was developed using recent sales in a particular neighborhood. The price (Y) of the house is based on the size (square footage = X) of the house. The model is:

$$Y = 13{,}473 + 37.65X$$

The coefficient of correlation for the model is 0.63.
a) Use the model to predict the selling price of a house that is 1,860 square feet.
b) An 1,860-square-foot house recently sold for $95,000. Explain why this is not what the model predicted.

c) If you were going to use multiple regression to develop such a model, what other quantitative variables might you include?
d) What is the value of the coefficient of determination in this problem?

• **4.49** Accountants at the Tucson firm, Larry Youdelman, CPAs, believed that several traveling executives were submitting unusually high travel vouchers when they returned from business trips. First, they took a sample of 200 vouchers submitted from the past year. Then they developed the following multiple-regression equation relating expected travel cost to number of days on the road (x_1) and distance traveled (x_2) in miles:

$$\hat{y} = \$90.00 + \$48.50x_1 + \$.40x_2$$

The coefficient of correlation computed was .68.
a) If Donna Battista returns from a 300-mile trip that took her out of town for 5 days, what is the expected amount she should claim as expenses?
b) Battista submitted a reimbursement request for $685. What should the accountant do?
c) Should any other variables be included? Which ones? Why?

•• **4.50** City government has collected the following data on annual sales tax collections and new car registrations:

ANNUAL SALES TAX COLLECTIONS (IN MILLIONS)	1.0	1.4	1.9	2.0	1.8	2.1	2.3
NEW CAR REGISTRATIONS (IN THOUSANDS)	10	12	15	16	14	17	20

Determine the following:
a) The least-squares regression equation.
b) Using the results of part (a), find the estimated sales tax collections if new car registrations total 22,000.
c) The coefficients of correlation and determination.

•• **4.51** Using the data in Problem 4.30, apply linear regression to study the relationship between the robbery rate and Dr. Fok's patient load. If the robbery rate increases to 131.2 in year 11, how many phobic patients will Dr. Fok treat? If the robbery rate drops to 90.6, what is the patient projection?

••• **4.52** Bus and subway ridership for the summer months in London, England, is believed to be tied heavily to the number of tourists visiting the city. During the past 12 years, the data on the next page have been obtained:

Light Thru My Lens Photography/Getty Images

YEAR (SUMMER MONTHS)	NUMBER OF TOURISTS (IN MILLIONS)	RIDERSHIP (IN MILLIONS)
1	7	1.5
2	2	1.0
3	6	1.3
4	4	1.5
5	14	2.5
6	15	2.7
7	16	2.4
8	12	2.0
9	14	2.7
10	20	4.4
11	15	3.4
12	7	1.7

a) Plot these data and decide if a linear model is reasonable.
b) Develop a regression relationship.
c) What is expected ridership if 10 million tourists visit London in a year?
d) Explain the predicted ridership if there are no tourists at all.
e) What is the standard error of the estimate?
f) What is the model's correlation coefficient and coefficient of determination?

•• **4.53** Thirteen students entered the business program at Sante Fe College 2 years ago. The following table indicates what each student scored on the high school SAT math exam and their grade-point averages (GPAs) after students were in the Sante Fe program for 2 years:

STUDENT	A	B	C	D	E	F	G
SAT SCORE	421	377	585	690	608	390	415
GPA	2.90	2.93	3.00	3.45	3.66	2.88	2.15
STUDENT	H	I	J	K	L	M	
SAT SCORE	481	729	501	613	709	366	
GPA	2.53	3.22	1.99	2.75	3.90	1.60	

a) Is there a meaningful relationship between SAT math scores and grades?
b) If a student scores a 350, what do you think his or her GPA will be?
c) What about a student who scores 800?

•• **4.54** Dave Fletcher, the general manager of North Carolina Engineering Corporation (NCEC), thinks that his firm's engineering services contracted to highway construction firms are directly related to the volume of highway construction business contracted with companies in his geographic area. He wonders if this is really so, and if it is, can this information help him plan his operations better by forecasting the quantity of his engineering services required by construction firms in each quarter of the year? The following table presents the sales of his services and total amounts of contracts for highway construction over the past eight quarters:

QUARTER	1	2	3	4	5	6	7	8
Sales of NCEC Services (in $ thousands)	8	10	15	9	12	13	12	16
Contracts Released (in $ thousands)	153	172	197	178	185	199	205	226

a) Using this data, develop a regression equation for predicting the level of demand of NCEC's services.

b) Determine the coefficient of correlation and the standard error of the estimate. Px

Additional problems **4.55-4.58** *are available in* MyOMLab.

Problems 4.59–4.61 relate to Monitoring and Controlling Forecasts

• • **4.59** Sales of tablet computers at Ted Glickman's electronics store in Washington, D.C., over the past 10 weeks are shown in the table below:

WEEK	DEMAND	WEEK	DEMAND
1	20	6	29
2	21	7	36
3	28	8	22
4	37	9	25
5	25	10	28

a) Forecast demand for each week, including week 10, using exponential smoothing with $\alpha = .5$ (initial forecast = 20).

b) Compute the MAD.

c) Compute the tracking signal. Px

• • • **4.60** The following are monthly actual and forecast demand levels for May through December for units of a product manufactured by the D. Bishop Company in Des Moines:

MONTH	ACTUAL DEMAND	FORECAST DEMAND
May	100	100
June	80	104
July	110	99
August	115	101
September	105	104
October	110	104
November	125	105
December	120	109

What is the value of the tracking signal as of the end of December?

Additional problem **4.61** *is available in* MyOMLab.

CASE STUDIES

Southwestern University: (B)*

Southwestern University (SWU), a large state college in Stephenville, Texas, enrolls close to 20,000 students. The school is a dominant force in the small city, with more students during fall and spring than permanent residents.

Always a football powerhouse, SWU is usually in the top 20 in college football rankings. Since the legendary Phil Flamm was hired as its head coach in 2009 (in hopes of reaching the elusive number 1 ranking), attendance at the five Saturday home games each year increased. Prior to Flamm's arrival, attendance generally averaged 25,000 to 29,000 per game. Season ticket sales bumped up by 10,000 just with the announcement of the new coach's arrival. Stephenville and SWU were ready to move to the big time!

Southwestern University Football Game Attendance, 2010–2015

	2010		2011		2012	
GAME	ATTENDEES	OPPONENT	ATTENDEES	OPPONENT	ATTENDEES	OPPONENT
1	34,200	Rice	36,100	Miami	35,900	USC
2[a]	39,800	Texas	40,200	Nebraska	46,500	Texas Tech
3	38,200	Duke	39,100	Ohio State	43,100	Alaska
4[b]	26,900	Arkansas	25,300	Nevada	27,900	Arizona
5	35,100	TCU	36,200	Boise State	39,200	Baylor

	2013		2014		2015	
GAME	ATTENDEES	OPPONENT	ATTENDEES	OPPONENT	ATTENDEES	OPPONENT
1	41,900	Arkansas	42,500	Indiana	46,900	LSU
2[a]	46,100	Missouri	48,200	North Texas	50,100	Texas
3	43,900	Florida	44,200	Texas A&M	45,900	South Florida
4[b]	30,100	Central Florida	33,900	Southern	36,300	Montana
5	40,500	LSU	47,800	Oklahoma	49,900	Arizona State

[a] Homecoming games.

[b] During the fourth week of each season, Stephenville hosted a hugely popular southwestern crafts festival. This event brought tens of thousands of tourists to the town, especially on weekends, and had an obvious negative impact on game attendance.

The immediate issue facing SWU, however, was not NCAA ranking. It was capacity. The existing SWU stadium, built in 1953, has seating for 54,000 fans. The following table indicates attendance at each game for the past 6 years.

One of Flamm's demands upon joining SWU had been a stadium expansion, or possibly even a new stadium. With attendance increasing, SWU administrators began to face the issue head-on. Flamm had wanted dormitories solely for his athletes in the stadium as an additional feature of any expansion.

SWU's president, Dr. Joel Wisner, decided it was time for his vice president of development to forecast when the existing stadium would "max out." The expansion was, in his mind, a given. But Wisner needed to know how long he could wait. He also sought a revenue projection, assuming an average ticket price of $50 in 2016 and a 5% increase each year in future prices.

Discussion Questions

1. Develop a forecasting model, justifying its selection over other techniques, and project attendance through 2017.
2. What revenues are to be expected in 2016 and 2017?
3. Discuss the school's options.

*This integrated case study runs throughout the text. Other issues facing Southwestern's football stadium include (A) managing the stadium project (Chapter 3); (C) quality of facilities (Chapter 6); (D) break-even analysis of food services (Supplement 7 Web site); (E) locating the new stadium (Chapter 8 Web site); (F) inventory planning of football programs (Chapter 12 Web site); and (G) scheduling of campus security officers/staff for game days (Chapter 13 Web site).

Forecasting Ticket Revenue for Orlando Magic Basketball Games

Video Case

For its first 2 decades of existence, the NBA's Orlando Magic basketball team set seat prices for its 41-game home schedule the same for each game. If a lower-deck seat sold for $150, that was the price charged, regardless of the opponent, day of the week, or time of the season. If an upper-deck seat sold for $10 in the first game of the year, it likewise sold for $10 for every game.

But when Anthony Perez, director of business strategy, finished his MBA at the University of Florida, he developed a valuable database of ticket sales. Analysis of the data led him to build a forecasting model he hoped would increase ticket revenue. Perez hypothesized that selling a ticket for similar seats should differ based on demand.

Studying individual sales of Magic tickets on the open Stub Hub marketplace during the prior season, Perez determined the additional potential sales revenue the Magic could have made had they charged prices the fans had proven they were willing to pay on Stub Hub. This became his dependent variable, y, in a multiple-regression model.

He also found that three variables would help him build the "true market" seat price for every game. With his model, it was possible that the same seat in the arena would have as many as seven different prices created at season onset—sometimes higher than expected on average and sometimes lower.

The major factors he found to be statistically significant in determining how high the demand for a game ticket, and hence, its price, would be were:

- The day of the week (x_1)
- A rating of how popular the opponent was (x_2)
- The time of the year (x_3)

For the day of the week, Perez found that Mondays were the least-favored game days (and he assigned them a value of 1). The rest of the weekdays increased in popularity, up to a Saturday game, which he rated a 6. Sundays and Fridays received 5 ratings, and holidays a 3 (refer to the footnote in Table 4.3).

His ratings of opponents, done just before the start of the season, were subjective and range from a low of 0 to a high of 8. A very high-rated team in that particular season may have had one or more superstars on its roster, or have won the NBA finals the prior season, making it a popular fan draw.

Fernando Medina

Finally, Perez believed that the NBA season could be divided into four periods in popularity:

- Early games (which he assigned 0 scores)
- Games during the Christmas season (assigned a 3)
- Games until the All-Star break (given a 2)
- Games leading into the play-offs (scored with a 3)

The first year Perez built his multiple-regression model, the dependent variable y, which was a "potential premium revenue score," yielded an $r^2 = .86$ with this equation:

$$y = 14{,}996 + 10{,}801x_1 + 23{,}397x_2 + 10{,}784x_3$$

Table 4.3 illustrates, for brevity in this case study, a sample of 12 games that year (out of the total 41 home game regular season), including the potential extra revenue per game (y) to be expected using the variable pricing model.

A leader in NBA variable pricing, the Orlando Magic have learned that regression analysis is indeed a profitable forecasting tool.

Discussion Questions*

1. Use the data in Table 4.3 to build a regression model with day of the week as the only independent variable.

TABLE 4.3 Data for Last Year's Magic Ticket Sales Pricing Model

TEAM	DATE*	DAY OF WEEK*	TIME OF YEAR	RATING OF OPPONENT	ADDITIONAL SALES POTENTIAL
Phoenix Suns	November 4	Wednesday	0	0	$12,331
Detroit Pistons	November 6	Friday	0	1	$29,004
Cleveland Cavaliers	November 11	Wednesday	0	6	$109,412
Miami Heat	November 25	Wednesday	0	3	$75,783
Houston Rockets	December 23	Wednesday	3	2	$42,557
Boston Celtics	January 28	Thursday	1	4	$120,212
New Orleans Pelicans	February 3	Monday	1	1	$20,459
L. A. Lakers	March 7	Sunday	2	8	$231,020
San Antonio Spurs	March 17	Wednesday	2	1	$28,455
Denver Nuggets	March 23	Sunday	2	1	$110,561
NY Knicks	April 9	Friday	3	0	$44,971
Philadelphia 76ers	April 14	Wednesday	3	1	$30,257

*Day of week rated as 1 = Monday, 2 = Tuesday, 3 = Wednesday, 4 = Thursday, 5 = Friday, 6 = Saturday, 5 = Sunday, 3 = holiday.

2. Use the data to build a model with rating of the opponent as the sole independent variable.
3. Using Perez's multiple-regression model, what would be the additional sales potential of a Thursday Miami Heat game played during the Christmas holiday?
4. What additional independent variables might you suggest to include in Perez's model?

*You may wish to view the video that accompanies this case before answering these questions.

Forecasting at Hard Rock Cafe

Video Case

With the growth of Hard Rock Cafe—from one pub in London in 1971 to more than 145 restaurants in 60 countries today—came a corporatewide demand for better forecasting. Hard Rock uses long-range forecasting in setting a capacity plan and intermediate-term forecasting for locking in contracts for leather goods (used in jackets) and for such food items as beef, chicken, and pork. Its short-term sales forecasts are conducted each month, by cafe, and then aggregated for a headquarters view.

The heart of the sales forecasting system is the point-of-sale (POS) system, which, in effect, captures transaction data on nearly every person who walks through a cafe's door. The sale of each entrée represents one customer; the entrée sales data are transmitted daily to the Orlando corporate headquarters' database. There, the financial team, headed by Todd Lindsey, begins the forecast process. Lindsey forecasts monthly guest counts, retail sales, banquet sales, and concert sales (if applicable) at each cafe. The general managers of individual cafes tap into the same database to prepare a daily forecast for their sites. A cafe manager pulls up prior years' sales for that day, adding information from the local Chamber of Commerce or Tourist Board on upcoming events such as a major convention, sporting event, or concert in the city where the cafe is located. The daily forecast is further broken into hourly sales, which drives employee scheduling. An hourly forecast of $5,500 in sales translates into 19 workstations, which are further broken down into a specific number of waitstaff, hosts, bartenders, and kitchen staff. Computerized scheduling software plugs in people based on their availability. Variances between forecast and actual sales are then examined to see why errors occurred.

Hard Rock doesn't limit its use of forecasting tools to sales. To evaluate managers and set bonuses, a 3-year weighted moving average is applied to cafe sales. If cafe general managers exceed their targets, a bonus is computed. Todd Lindsey, at corporate headquarters, applies weights of 40% to the most recent year's sales, 40% to the year before, and 20% to sales 2 years ago in reaching his moving average.

An even more sophisticated application of statistics is found in Hard Rock's menu planning. Using multiple regression, managers can compute the impact on demand of other menu items if the price of one item is changed. For example, if the price of a cheeseburger increases from $7.99 to $8.99, Hard Rock can predict the effect this will have on sales of chicken sandwiches, pork sandwiches, and salads. Managers do the same analysis on menu placement, with the center section driving higher sales volumes. When an item such as a hamburger is moved off the center to one of the side flaps, the corresponding effect on related items, say french fries, is determined.

HARD ROCK'S MOSCOW CAFE[a]

MONTH	1	2	3	4	5	6	7	8	9	10
Guest count (in thousands)	21	24	27	32	29	37	43	43	54	66
Advertising (in $ thousand)	14	17	25	25	35	35	45	50	60	60

[a] These figures are used for purposes of this case study.

Discussion Questions*

1. Describe three different forecasting applications at Hard Rock. Name three other areas in which you think Hard Rock could use forecasting models.
2. What is the role of the POS system in forecasting at Hard Rock?
3. Justify the use of the weighting system used for evaluating managers for annual bonuses.
4. Name several variables besides those mentioned in the case that could be used as good predictors of daily sales in each cafe.
5. At Hard Rock's Moscow restaurant, the manager is trying to evaluate how a new advertising campaign affects guest counts. Using data for the past 10 months (see the table), develop a least-squares regression relationship and then forecast the expected guest count when advertising is $65,000.

*You may wish to view the video that accompanies this case before answering these questions.

- **Additional Case Studies:** Visit MyOMLab for these free case studies:
  North-South Airlines: Reflects the merger of two airlines and addresses their maintenance costs.
  Digital Cell Phone, Inc.: Uses regression analysis and seasonality to forecast demand at a cell phone manufacturer.

Endnotes

1. For a good review of statistical terms, refer to Tutorial 1, "Statistical Review for Managers," in MyOMLab.
2. When the sample size is large ($n > 30$), the prediction interval value of y can be computed using normal tables. When the number of observations is small, the t-distribution is appropriate. See D. Groebner et al., *Business Statistics*, 9th ed. (Upper Saddle River, NJ: Prentice Hall, 2014).
3. To prove these three percentages to yourself, just set up a normal curve for ± 1.6 standard deviations (z-values). Using the normal table in Appendix I, you find that the area under the curve is .89. This represents ± 2 MADs. Likewise, ± 3 MADs $= \pm 2.4$ standard deviations encompass 98% of the area, and so on for ± 4 MADs.
4. Bernard T. Smith, *Focus Forecasting: Computer Techniques for Inventory Control* (Boston: CBI Publishing, 1978).

Chapter 4 *Rapid* Review

Main Heading	Review Material	MyOMLab
WHAT IS FORECASTING? (pp. 108–109)	■ **Forecasting**—The art and science of predicting future events. ■ **Economic forecasts**—Planning indicators that are valuable in helping organizations prepare medium- to long-range forecasts. ■ **Technological forecasts**—Long-term forecasts concerned with the rates of technological progress. ■ **Demand forecasts**—Projections of a company's sales for each time period in the planning horizon.	Concept Questions: 1.1–1.4
THE STRATEGIC IMPORTANCE OF FORECASTING (pp. 109–110)	*The forecast is the only estimate of demand until actual demand becomes known.* Forecasts of demand drive decisions in many areas, including: human resources, capacity, and supply chain management.	Concept Questions: 2.1–2.3
SEVEN STEPS IN THE FORECASTING SYSTEM (pp. 110–111)	■ Forecasting follows seven basic steps: (1) Determine the use of the forecast; (2) Select the items to be forecasted; (3) Determine the time horizon of the forecast; (4) Select the forecasting model(s); (5) Gather the data needed to make the forecast; (6) Make the forecast; (7) Validate and implement the results.	Concept Questions: 3.1–3.4
FORECASTING APPROACHES (pp. 111–112)	■ **Quantitative forecasts**—Forecasts that employ mathematical modeling to forecast demand. ■ **Qualitative forecast**—Forecasts that incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system. ■ **Jury of executive opinion**—Takes the opinion of a small group of high-level managers and results in a group estimate of demand. ■ **Delphi method**—Uses an interactive group process that allows experts to make forecasts. ■ **Sales force composite**—Based on salespersons' estimates of expected sales. ■ **Market survey**—Solicits input from customers or potential customers regarding future purchasing plans. ■ **Time series**—Uses a series of past data points to make a forecast.	Concept Questions: 4.1–4.4
TIME-SERIES FORECASTING (pp. 112–131)	■ **Naive approach**—Assumes that demand in the next period is equal to demand in the most recent period. ■ **Moving average**—Uses an average of the n most recent periods of data to forecast the next period. $$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n} \quad (4\text{-}1)$$ $$\text{Weighted moving average} = \frac{\sum((\text{Weight for period } n)(\text{Demand in period } n))}{\sum \text{Weights}} \quad (4\text{-}2)$$ ■ **Exponential smoothing**—A weighted-moving-average forecasting technique in which data points are weighted by an exponential function. ■ **Smoothing constant**—The weighting factor, α, used in an exponential smoothing forecast, a number between 0 and 1. Exponential smoothing formula: $$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (4\text{-}4)$$ ■ **Mean absolute deviation (MAD)**—A measure of the overall forecast error for a model. $$\text{MAD} = \frac{\sum \lvert \text{Actual} - \text{Forecast} \rvert}{n} \quad (4\text{-}5)$$ ■ **Mean squared error (MSE)**—The average of the squared differences between the forecast and observed values. $$\text{MSE} = \frac{\sum (\text{Forecast errors})^2}{n} \quad (4\text{-}6)$$ ■ **Mean absolute percent error (MAPE)**—The average of the absolute differences between the forecast and actual values, expressed as a percentage of actual values. $$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \lvert \text{Actual}_i - \text{Forecast}_i \rvert / \text{Actual}_i}{n} \quad (4\text{-}7)$$	Concept Questions: 5.1–5.4 Problems: 4.1–4.42 Virtual Office Hours for Solved Problems: 4.1–4.4 **ACTIVE MODELS 4.1–4.4**

Chapter 4 *Rapid* Review *continued*

Main Heading	Review Material	MyOMLab
	Exponential smoothing with trend adjustment Forecast including trend (FIT_t) = Exponentially smoothed forecast average (F_t) + Exponentially smoothed trend (T_t) (4-8) ■ **Trend projection**—A time-series forecasting method that fits a trend line to a series of historical data points and then projects the line into the future for forecasts. Trend projection and regression analysis $\hat{y} = a + bx$, where $b = \dfrac{\sum xy - n\overline{x}\overline{y}}{\sum x^2 - n\overline{x}^2}$ and $a = \overline{y} - b\overline{x}$ (4-11), (4-12), (4-13) ■ **Seasonal variations**—Regular upward or downward movements in a time series that tie to recurring events. ■ **Cycles**—Patterns in the data that occur every several years.	Virtual Office Hours for Solved Problems: 4.5–4.6
ASSOCIATIVE FORECASTING METHODS: REGRESSION AND CORRELATION ANALYSIS (pp. 131–137)	■ **Linear-regression analysis**—A straight-line mathematical model to describe the functional relationships between independent and dependent variables. ■ **Standard error of the estimate**—A measure of variability around the regression line. ■ **Coefficient of correlation**—A measure of the strength of the relationship between two variables. ■ **Coefficient of determination**—A measure of the amount of variation in the dependent variable about its mean that is explained by the regression equation. ■ **Multiple regression**—An associative forecasting method with > 1 independent variable. Multiple regression forecast: $\hat{y} = a + b_1x_1 + b_2x_2$ (4-17)	Concept Questions: 6.1–6.4 Problems: 4.43-4.58 **VIDEO 4.1** Forecasting Ticket Revenue for Orlando Magic Basketball Games Virtual Office Hours for Solved Problems: 4.7–4.8
MONITORING AND CONTROLLING FORECASTS (pp. 138–140)	■ **Tracking signal**—A measurement of how well the forecast is predicting actual values. $\text{Tracking signal} = \dfrac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\text{MAD}}$ (4-18) ■ **Bias**—A forecast that is consistently higher or lower than actual values of a time series. ■ **Adaptive smoothing**—An approach to exponential smoothing forecasting in which the smoothing constant is automatically changed to keep errors to a minimum. ■ **Focus forecasting**—Forecasting that tries a variety of computer models and selects the best one for a particular application.	Concept Questions: 7.1–7.4 Problems: 4.59–4.61
FORECASTING IN THE SERVICE SECTOR (pp. 140–141)	Service-sector forecasting may require good short-term demand records, even per 15-minute intervals. Demand during holidays or specific weather events may also need to be tracked.	Concept Question: 8.1 **VIDEO 4.2** Forecasting at Hard Rock Cafe

Self Test

■ **Before taking the self-test,** refer to the learning objectives listed at the beginning of the chapter and the key terms listed at the end of the chapter.

LO 4.1 Forecasting time horizons include:
- **a)** long range.
- **b)** medium range.
- **c)** short range.
- **d)** all of the above.

LO 4.2 Qualitative methods of forecasting include:
- **a)** sales force composite.
- **b)** jury of executive opinion.
- **c)** consumer market survey.
- **d)** exponential smoothing.
- **e)** all except (d).

LO 4.3 The difference between a *moving-average* model and an *exponential smoothing* model is that ______.

LO 4.4 Three popular measures of forecast accuracy are:
- **a)** total error, average error, and mean error.
- **b)** average error, median error, and maximum error.
- **c)** median error, minimum error, and maximum absolute error.
- **d)** mean absolute deviation, mean squared error, and mean absolute percent error.

LO 4.5 Average demand for iPods in the Rome, Italy, Apple store is 800 units per month. The May monthly index is 1.25. What is the seasonally adjusted sales forecast for May?
- **a)** 640 units
- **b)** 798.75 units
- **c)** 800 units
- **d)** 1,000 units
- **e)** cannot be calculated with the information given

LO 4.6 The main difference between simple and multiple regression is ______.

LO 4.7 The tracking signal is the:
- **a)** standard error of the estimate.
- **b)** cumulative error.
- **c)** mean absolute deviation (MAD).
- **d)** ratio of the cumulative error to MAD.
- **e)** mean absolute percent error (MAPE).

Answers: LO 4.1. d; LO 4.2. e; LO 4.3. exponential smoothing is a weighted moving-average model in which all prior values are weighted with a set of exponentially declining weights; LO 4.4. d; LO 4.5. d; LO 4.6. simple regression has only one independent variable; LO 4.7. d.